Physica A 393 (2014) 261–267

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

The size of compact extra dimensions from blackbody radiation laws

Ramaton Ramos [a], Henrique Boschi-Filho [b,*]

[a] Centro Brasileiro de Pesquisas Físicas, Rua Dr. Xavier Sigaud 150, Urca, RJ 22290-180, Brazil

[b] Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, RJ 21941-972, Brazil

HIGHLIGHTS

- We generalize the blackbody radiation laws to include compact extra dimensions.
- We consider different dimensions $D = 10$ and $D = 11$, motivated by string/M-theory.
- We estimate the size of extra dimensions comparing our results with experimental data.

ARTICLE INFO

Article history:
Received 9 June 2013
Received in revised form 30 August 2013
Available online 27 September 2013

Keywords:
Blackbody radiation
Stefan–Boltzmann law
Wien's law
Compact extra dimensions

ABSTRACT

In this work we generalize the Stefan–Boltzmann and Wien's displacement laws for a D-dimensional manifold composed by 4 non-compact dimensions and $D - 4$ compact dimensions, $\mathbb{R}^{1,3} \times \mathbb{T}^{D-4}$. The electromagnetic field is assumed to pervade all compact and non-compact dimensions. In particular, the total radiated power becomes $R(T) = \sigma_B T^4 + \sigma_D(a)\, T^D$, where a is the size of the compact extra dimensions. For $D = 10$, predicted from String Theory, and $D = 11$, from M-Theory, the outcomes agree with available experimental data for a as high as $2 \cdot 10^{-7}$ m.

1. Introduction

Interesting proposals to solve open problems in particle physics, like the Hierarchy issue [1,2], and in cosmology, like the dark matter and dark energy, involve spacetimes with dimensions different from the usually accepted four. The original proposal of a model with one extra dimension was done by Kaluza and Klein in the 1920's to unify gravity, described by the general theory of relativity, with Maxwell's electromagnetism. In their model, the fifth dimension is compactified on a circle of a given radius, while the other four non-compact dimensions are identified with our usual spacetime [3].

String theory is presently the most viable candidate for unifying gravity with the other fundamental interactions described by quantum field theories and requires a 9 + 1 dimensional spacetime. String theory is also related to M-theory defined in 10 + 1 dimensional spacetime [4]. Usually, the extra dimensions are supposed to be compact, with a compactification parameter that might be related to the Planck scale.

About a decade ago, different models have been proposed to deal with the size of extra dimensions, keeping the extra dimensions compact [5,6] or non-compact [7]. Many predictions of these and other models are being tested in the ongoing LHC experiments [8,9], with hadronic beams colliding now at 7 TeV and in the forthcoming years at 14 TeV.

* Corresponding author. Tel.: +55 21 2562 7919; fax: +55 21 2562 7368.
E-mail addresses: ramaton@gmail.com (R. Ramos), boschi@if.ufrj.br (H. Boschi-Filho).

0378-4371/$ – see front matter
http://dx.doi.org/10.1016/j.physa.2013.09.052

Another way to test the predictions of string/M-theory is to look at the physics of low energy processes. It is expected that, in some way, string/M-theory shall reproduce the physics that one can observe in scales much lower than the Planck scale.

Blackbody radiation is very well described by Planck's law which implies, and explains, the Stefan–Boltzmann and Wien laws. In particular, the Stefan–Boltzmann law predicts that the electromagnetic radiated power of a blackbody is $R(T) = \sigma_B T^4$, where σ_B is the Stefan–Boltzmann constant.

Recently, it was pointed out that the blackbody radiation laws should be corrected if the radiation is confined into a cavity [10]. On the other side, it has been noticed that the blackbody radiation laws should depend on the (flat and non-compact) spacetime dimensions D [11,12] such that the Stefan–Boltzmann law would be modified to $R(T) = \sigma_D T^D$ with $\sigma_D =$ constant, in contrast with observed data.

The main result of this work is the blackbody radiated power $R(T) = \sigma_B T^4 + \sigma_D(a)\, T^D$ for a spacetime with four non-compact dimensions and $D - 4$ compact dimensions. This outcome reproduces, at low temperatures, the well-known Stefan–Boltzmann law and also the calculated relation for a D-dimensional spacetime [11,12] in the high temperature regime, as we are going to show in the following sections.

This article is organized as follows: In Section 2 we present a brief account of the blackbody concept and introduce our approach. Section 3 is routed to the generalization of the Stefan–Boltzmann and Wien's displacement laws along with their peculiarities for a $\mathbb{R}^{1,3} \times \mathbb{T}^{D-4}$ spacetime. In Section 4 we consider recent high-temperature blackbody experiments in order to calculate bounds on the size of the considered compact dimensions for String theory ($D = 10$) and M-theory ($D = 11$). The final section is devoted to general comments and closing remarks.

2. Cavity radiation

A blackbody is defined as a body with a rich energy spectrum, capable of exciting all frequencies of light by thermalization. As consequence, all blackbodies emit thermal radiation with the same spectrum. For a current review on the matter, see Ref. [13].

In order to study its properties one conveniently takes a small bidimensional orifice connecting an isothermal enclosure to its outside as a blackbody surface (a more technical and conceptual discussion on approximating a blackbody for a blackbox can be found in Ref. [14]). Here we consider that the blackbody is immersed in a D-dimensional spacetime, $\mathbb{R}^{1,3} \times \mathbb{T}^{D-4}$.

The electromagnetic radiation inside the enclosure is assumed to be composed of standing waves. Choosing a system of orthogonal coordinates with origin at one of the enclosure's vertices, we take for the sake of simplicity [15] ℓ as the length of the edges parallel to the non-compact axes x_i ($i = 1, 2, 3$) and a as the length of the edges related to the compact axes x_j ($j = 4, \ldots, D - 1$), so the i-th and j-th components of the electric field ($l = i, j$) are given by

$$E_l(x_l, t) = E_{0,l} \sin(\mathrm{k}_l x_l)\, \mathrm{e}^{-\mathrm{i}\omega t}. \tag{1}$$

The i-th electric field components satisfy Dirichlet boundary conditions $E_i(0, t) = E_i(\ell, t) = 0$, for which there is no surface currents on the isothermal cavity walls, while the j-th components satisfy periodic boundary conditions $E_j(x_j, t) = E_j(x_j + a, t)$, regarding the compactness of its corresponding dimensions. Thus

$$\mathrm{k}_i = \frac{\pi}{\ell}\, n_i, \qquad \mathrm{k}_j = \frac{2\pi}{a}\, n_j, \tag{2}$$

where $n_i, n_j = 0, 1, 2, 3, \ldots$ represent the possible modes of vibration.

Considering the standing waves like components of a plain electromagnetic one with wave-vector $\vec{\mathrm{k}}$ and since the frequency $\nu = |\vec{\mathrm{k}}|c/2\pi$, we get for each mode (n_i, n_j)

$$\nu_{ij} \equiv \nu(n_i, n_j) = \frac{c}{a}\sqrt{\frac{a^2}{4\ell^2}\sum_{i=1}^{3} {n_i}^2 + \sum_{j=4}^{D-1} {n_j}^2}. \tag{3}$$

Making use of Bose–Einstein statistical prescription and accounting two helicity states related to the propagative aspect of the photons associated with the standing waves, since we do not consider polarization along the compact dimensions, the energy density inside the isothermal enclosure maintained at temperature T is

$$\rho(T) = \frac{2}{V}\sum_{n_i, n_j} \frac{h\nu_{ij}}{\mathrm{e}^{h\nu_{ij}/kT} - 1}, \tag{4}$$

with $V = \ell^3$, the 3-dimensional cavity volume.

The energy density is proportional to the radiancy $R(T)$, the energy rate per unit area of the orifice. Considering geometric factors for which the emanated power propagates only through the 3 non-compact dimensions, since the other ones are compact, one gets

$$R(T) = \frac{c}{4}\,\rho(T). \tag{5}$$