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Cluster lag synchronisation in community networks via linear pinning control with local intermittent effect

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HIGHLIGHTS

- Cluster lag synchronization in community networks without or with delay is considered.
- We use both pinning control and intermittent control schemes, in view of their lower cost and more convenient implementation.
- Adaptive coupling strength method is adopted to make the coupling strength as small as possible.
- Our proposed control schemes are more applicable technically in practical problems.
- Several linear pinning controllers are designed to achieve the synchronization.

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ABSTRACT

In this article, cluster lag synchronisation in community networks without or with delay is considered. Several linear pinning controllers have been designed to achieve the synchronisation. In the designed controllers, the feedback control item is designed with intermittent effect. Both community networks with identical nodes and non-identical nodes are investigated. Based on the Lyapunov stability theory, several synchronisation criteria are derived. Some numerical examples are provided to illustrate the effectiveness of the obtained theoretical results.

1. Introduction

Community networks, as a class of special network models, are ubiquitous in real complex systems, such as social and biological networks [1], school friendship networks [2]. congressional cosponsorship networks [3] and so on. Generally, in community networks, there is a higher density of connections in each community and a lower density of connections between any two communities. The local dynamics of nodes in a community may be identical; however, in many real networks, they are non-identical: the nodes in the same community may have the same node dynamics, while the nodes in different communities may have different node dynamics. Therefore, in many situations, community networks with non-identical nodes can describe the real world better.

Synchronisation, as a typical collective dynamical behaviour of complex networks, has been widely studied [4–31]. In community networks, especially those with non-identical nodes, the nodes belonging to different communities usually tend to synchronise with different states, that is, to achieve the cluster synchronisation. In Refs. [4–7], cluster synchronisation in community networks with non-identical nodes was studied, and several sufficient conditions for synchronisation were obtained analytically.

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In many practical situations, there are usually propagation delays, which have been observed in lasers, neuronal models, electronic circuits and so on [10–12]. Therefore, lag synchronisation of a complex network has been taken into consideration [13–18], which has already presented possible and potential technological applications in many fields, such as information transformation among neurons, secure communication and electronic implementation of dynamical systems. Furthermore, if information transform in a community network, then the propagation delays are unavoidable and should be considered. Due to the properties of community networks, the propagation delays in the same community may be identical, while in different communities they may be non-identical. That is, in a community network, different communities may achieve lag synchronisation with different time delays; we call this cluster lag synchronisation (CLS). It is a new type of synchronisation deserving a detailed study.

Furthermore, many real complex networks cannot synchronise themselves or synchronise with the desired orbits. Therefore, proper controllers should be designed to achieve the goals by adopting some control schemes, such as adaptive control [19], feedback control [20], observer-based control [21], impulsive control [22], intermittent control [6,23–27], pinning control [6,13,26–31] and so on. The intermittent control scheme, which can be regarded as a transition from continuous-time control to impulsive control, is extensively adopted to design controllers due to the lower control cost and convenient implementation. For a large-scale network, the pinning control method is welcome in that it only needs to control a small fraction of all the nodes and reduce the control cost heavily. In a study conducted by Yu et al. [29], pinning synchronisation of complex dynamical networks was considered and some interesting and useful results were obtained. Specially, in the study conducted by Liu et al. [6], the intermittent pinning control method was adopted to investigate the cluster synchronisation in directed networks and a centralised adaptive intermittent control introduced and theoretical analysis provided. Naturally, how to adopt intermittent pinning control method to study the CLS problem becomes an important issue.

Motivated by the above discussions, this article investigates CLS in community networks without or with delay using the intermittent pinning control method. To achieve the synchronisation, several linear pinning controllers with local intermittent effect are designed. First, the CLSI n community networks with identical nodes and non-identical nodes are discussed, respectively. Second, the CLS in delayed community networks with identical nodes and non-identical nodes are discussed as well. According to Lyapunov stability theory, the sufficient conditions for achieving CLS are obtained analytically.

This article is organised as follows. Section 2 introduces the network models and some preliminaries. Section 3 considers the CLS in community networks via designing linear pinning controllers with intermittent effect. Section 4 provides several numerical simulations to verify the correctness and effectiveness of the derived results. Section 5 concludes this article.

Notation. Throughout this article, for a symmetric matrix P, the notation $P > 0$ ($P < 0$) means that the matrix P is positive definite (negative definite). I_N denotes the $N \times N$ identity matrix. Q^T denotes the transposed matrix of Q.

2. Model description and preliminaries

Consider a complex network consisting of N nodes and l communities with $2 \le l < N$, which can be described by

$$\dot{x}_i(t) = f(x_i(t)) + \varepsilon \sum_{k=1}^{l} \sum_{j \in V_k} c_{ij} \Gamma x_j(t), \tag{1}$$

where $i = 1, 2, \ldots, N$, $x_i(t) = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in R^n$ is the state variable of node i, $\varepsilon > 0$ is the coupling strength and $\Gamma = \mathrm{diag}(\gamma_1, \gamma_2, \ldots, \gamma_n)$ is the inner coupling matrix. The matrix $C = (c_{ij})_{N\times N}$ is the zero-row-sum outer coupling matrix, which denotes the network topology and is defined as follows: if there is a connection between node i and node j ($i \neq j$), then $c_{ij} > 0$, otherwise, $c_{ij} = 0$; V_k ($k = 1, 2, \ldots, l$) denotes the set of all nodes belonging to the kth community.

Let the error

$$e_i(t) = x_i(t) - s(t - \tau_{\varphi_i}), \quad i = 1, 2, \ldots, N, \tag{2}$$

where $s(t)$ is a solution of an isolated node, that is, $\dot{s}(t) = f(s(t))$, and $s(t)$ may be an equilibrium point, a periodic orbit or even a chaotic orbit. The function φ is defined as $\varphi : \{1, 2, \ldots, N\} \to \{1, 2, \ldots, l\}$; if a node $i \in V_k$, then $\varphi_i = k$.

In the subsequent discussions, we always assume that $s(t)$ cannot be an equilibrium point, and if $\varphi_i \neq \varphi_j$, then $\tau_{\varphi_i} \neq \tau_{\varphi_j}$.

Definition 1. Network (1) is said to achieve CLS with respect to the time delays τ_{φ_i}, if $\lim_{t\to\infty} ||x_i(t) - s(t - \tau_{\varphi_i})|| = 0$, $i = 1, 2, \ldots, N$.

For achieving the CLS of network (1), some linear controllers are needed. The controlled network can be written as

$$\dot{x}_i(t) = f(x_i(t)) + \varepsilon \sum_{j=1}^{N} c_{ij} \Gamma x_j(t) + u_i(t), \tag{3}$$

where $u_i(t)$ ($i = 1, 2, \ldots, N$) are the controllers to be designed later.