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Cluster synchronization in directed networks of non-identical systems with noises via random pinning control

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HIGHLIGHTS

- Mean square cluster synchronization in directed networks is studied.
- The networks consist of non-identical systems infected by communication noises.
- Bernoulli stochastic variables are used to describe the occurrences of control.
- Sufficient conditions containing two aspects are proposed.

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ABSTRACT

This paper is concerned with the issue of mean square cluster synchronization in directed networks, which consist of non-identical nodes infected by communication noises. The pinning control method is employed in designing controllers for guaranteeing cluster synchronization, meanwhile, all the controllers are supposed to occur with different probabilities by introducing the Bernoulli stochastic variables. Based on the Lyapunov stability theory and the stochastic theory, the sufficient synchronization conditions are derived and proved theoretically, which are mainly for the controllers to be designed and the noise intensities. Finally, some numerical examples are presented to demonstrate the effectiveness of the results.

1. Introduction

Complex networks exist everywhere in the real world, such as the World Wide Web, the biological neural networks, and the power grids [1–3]. As an important and interesting collective behavior of networks, synchronization has been studied extensively. So far many different kinds of synchronization including complete synchronization, generalized synchronization, phase synchronization and cluster synchronization have been introduced [4–9]. Moreover, study about synchronization in time-varying complex networks was also reported [10]. Cluster synchronization means that the nodes in a network split into subgroups called clusters, such that the nodes belonging to the same cluster rather than different clusters can realize complete synchronization. The importance of cluster synchronization has been found in biological science [11,12] and communication engineering [13,14].

In the case where the whole network cannot synchronize by its intrinsic structure, some control schemes may be designed to drive the network to synchronization. However, it is usually difficult to control all nodes in a large-scale network,

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then pinning control is an effective strategy, which implies that we just need to control a fraction of the nodes in order to synchronize the whole network. Nowadays, by means of pinning control, the problem of controlling a general network towards an assigned synchronous evolution was studied [15,16]. In addition, Refs. [17–20] investigated cluster synchronization via the pinning control method. Recently, Liu et al. considered the controllability of an arbitrary complex directed network, and identified the set of driver nodes with time-dependent control that can guide the system's entire dynamics [21], which further explains the significance of pinning control. On the other hand, controlled plants may suffer from disturbances from random failures, packet dropouts, sudden environmental changes and so on, therefore randomly occurring control is more realistic compared with continuous control. To the best of our knowledge, synchronization in networks by adopting random control scheme was given in Refs. [22,23], but research on cluster synchronization via randomly occurring control has received little attention until now.

It is well known that noise is unavoidable in practical environment, then it is necessary to study cluster synchronization for dynamic systems with noises. There are some related works, such as cluster synchronization in a neural network with noises was investigated [24], Wu et al. [25] gave the numerical examples about cluster synchronization of a BA scale-free network under noise disturbances, and Ref. [26] considered cluster synchronization in a network with delays and stochastic perturbation based on adaptive control method.

Motivated by the above discussion, this paper further studies the problem of mean square cluster synchronization in directed networks. Different from most of the existing literature (e.g., Refs. [17–20,24–26]), the random pinning control strategy is utilized, and all the nodes in the network are supposed to be affected by communication noises. In the network considered here, the nodes belonging to different clusters can be non-identical, and the coupling matrix of the network does not need to be symmetric or irreducible. In addition, in many practical systems it is unrealistic to assume all nodes to be cooperative. Consequently, similar to Refs. [18,19], competitive couplings besides cooperative couplings are introduced into the network. Obviously, cooperative and competitive dynamic systems are more realistic in practice.

The main contributions of this work can be summarized as follows. The first one is that the random pinning control (i.e., the pinning control occurring with probability) is adopted in designing controllers, which is an economic and realistic scheme for cluster synchronization; the second one is that both communication noises and competitive couplings are taken into account for the network, which may be more consistent with the real-world case.

The rest of the paper is organized as follows. In Section 2, the problem is formulated and some useful definitions and lemmas are given. Theoretical results for mean square cluster synchronization are derived in Section 3. In Section 4, some numerical examples are shown to illustrate the analysis. Finally, concluding remarks are presented and discussed.

The following notations are given which will be used throughout this paper: let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}^n$ the n-dimensional Euclidean space and $\mathbb{R}^{n_1\times n_2}$ the set of $n_1 \times n_2$ real matrices. $\mathbf{0}_n$ denotes the n-dimensional vector of zeros. $\mathbf{1}_n$ denotes the n-dimensional vector of ones. $\boldsymbol{I}_n$ denotes the $n \times n$ identity matrix. $\boldsymbol{diag}\{a_1, a_2, \dots, a_n\}$ denotes the $n \times n$ diagonal matrix with elements $a_1, a_2, \dots, a_n \in \mathbb{R}$ on the diagonal. The superscript "T" denotes matrix transposition. $\boldsymbol{X} > 0$ ($\boldsymbol{X} < 0$) means the symmetric matrix $\boldsymbol{X}$ is positive (negative) definite. $\|\cdot\|$ indicates the Euclidean norm. $|\cdot|$ stands for the absolute value. $tr(\cdot)$ indicates the trace of a square matrix. $\mathbb{E}(\cdot)$ indicates the mathematical expectation of a random variable. $\otimes$ represents the Kronecker product.

2. Problem formulation

Consider a general model of directed networks consisting of N non-identical nodes described as follows:

$$\dot{\boldsymbol{x}}_i(t) = \boldsymbol{f}_i(\boldsymbol{x}_i(t)) + c\sum_{j=1}^{N} a_{ij}\boldsymbol{y}_{ij}(t), \quad i = 1, 2, \dots, N, \tag{1}$$

where $\boldsymbol{x}_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in \mathbb{R}^n$ denotes the state vector of the ith node. $\boldsymbol{f}_i$: $\mathbb{R}^n \to \mathbb{R}^n$ is a continuous vector function representing the activity of the ith system, $c > 0$ stands for the coupling strength, and topology of the network is represented by the coupling matrix $\boldsymbol{A} = (a_{ij}) \in \mathbb{R}^{N\times N}$, where $a_{ij} \neq 0$ if the node i receives direct information from the node j, otherwise $a_{ij} = 0$. $\boldsymbol{y}_{ij}(t)$ stands for the transmission information from the node j to the node i, which is impacted by noises and defined as $\boldsymbol{y}_{ij}(t) = \boldsymbol{x}_j(t) + (\sigma_{ij}(t) \otimes \mathbf{1}_n)\xi_{ij}(t)$, where $\sigma_{ij}(t) \geq 0$ indicates the noise intensity, and $\xi_{ij}(t)$ denotes a standard white noise existing in the transmission channel. Since the time derivative of a Wiener process (Brownian motion) is a white noise process in the stochastic theory, the network (1) can be rewritten as:

$$\mathrm{d}\boldsymbol{x}_i(t) = \left[\boldsymbol{f}_i(\boldsymbol{x}_i(t)) + c\sum_{j=1}^{N} a_{ij}\boldsymbol{x}_j(t)\right]\mathrm{d}t + c\sum_{j=1}^{N} a_{ij}(\sigma_{ij}(t) \otimes \mathbf{1}_n)\,\mathrm{d}w_{ij}(t), \quad i = 1, 2, \dots, N, \tag{2}$$

where $w_{ij}(t)$ is a scalar Brownian motion defined on $(\Omega, \mathcal{F}, P)$, which satisfies $\mathbb{E}[\mathrm{d}w_{ij}(t)] = 0$ and $\mathbb{E}\{[\mathrm{d}w_{ij}(t)]^2\} = \mathrm{d}t$.

Remark 1. If a_{ij} is changed into $a_{ij}(t)$, then the topology of the network will be switching or time-varying. Recently, some results concerning consensus in switching networks have been presented, wherein the concept of consensus is similar to that of synchronization. For example, supposing that the network has a spanning tree or is jointly connected, then based