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Analysis of the distribution of the order parameter of synthetic seismicity generated by a simple spring–block system with asperities

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HIGHLIGHTS

- Synthetic seismicity is generated by an experimental spring–block system with asperities.
- The order parameter for the seismicity is formulated in the natural time domain.
- The order parameter in NTD was applied to the synthetic seismicity.
- Is pointed out the scaled probability density of the order parameter collapses into a universal curve.

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ABSTRACT

By analyzing the synthetic seismicity generated by a simple spring–block system with asperities, the probability distribution of its order parameter is investigated. The spring–block system mimics the interaction between tectonic plates, whose asperities are given by sandpapers of different grades. The analysis is performed in the natural time domain, which was shown to be an important tool to obtain relevant information hidden in the time series of complex systems. We find that the scaled probability density functions of the order parameter almost collapse onto a single curve with non-Gaussian tails. This indicates that the system is in a critical state similar to what is reported for several real seismic catalogues and numerical simulations of different complex systems.

1. Introduction

In recent years the interest in the analysis of complex systems has been growing, involving different research areas. In particular, systems whose critical point corresponds to second-order phase transitions are considered of crucial importance in statistical physics due to the large variety of applications [1]. In order to describe the statistical features of such systems in a critical state, it is necessary to define an order parameter (OP) that is expected to be characterized by non-Gaussian distributions. However, except few cases, the mathematical form of the probability density function (PDF) of the OP is not known [2,3]. Therefore, the research aiming at better defining the type of fluctuations of the OP at criticality is challenging.

Systems like the Ising, Potts, *XY* models, turbulent fluxes [4–7] are characterized by the scaled PDF of the normalized OP collapsing onto a non-Gaussian universal curve, and this is a direct signature of criticality shown by large fluctuations below the mean towards small values of the OP.

A seismic process is a system with complex correlations in time, space and magnitude [8], and the power-law relationships involving such parameters are widely recognized to be a signature of the proximity of the system to a critical state.

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The scaled PDF of the OP for seismicity exhibits features similar to those observed in several equilibrium critical phenomena and in nonequilibrium systems [9].

An OP for real seismicity was introduced in Ref. [8] and reviewed in Ref. [10] in the context of the natural time domain (NTD). A direct physical interconnection between this OP of seismicity and precursory electrical signals [11–14] has been recently identified by analyzing real seismic data [15].

2. Natural time analysis

It has been shown that some unique dynamic features hidden behind can be derived from the time series of complex systems, if we analyze them in terms of natural time χ. The Natural Time Domain (NTD) approach was developed in Ref. [16] and in Ref. [17]. Given a sequence of N events, the natural time can be defined as $\chi_k = k/N$ indicating an index of occurrence of the k-th event. In natural time domain, the pair (χ_k, Q_k) is considered, where Q_k denotes a quantity proportional to the energy associated with the k-th event. In doing so, we ignore the time intervals between consecutive events, but preserve their order and energy Q_k. In the case of seismicity, Q_k is the seismic moment of the k-th earthquake, proportional to the seismic energy released E_k. Instead of Q_k it is convenient to consider the quantity

$$p_k = \frac{E_k}{\sum_{i=1}^{N} E_i} \tag{1}$$

$$\sum_{i=1}^{N} p_k = 1 \tag{2}$$

where p_k is the probability of the k-th energy normalized to the total energy. Hence the natural time analysis is performed on the pair (χ_k, p_k).

For seismicity one order parameter defined by [10,16] in NTD is

$$k_1 = \sum_{i=1}^{N} p_i \chi_i^2 - \left(\sum_{i=1}^{N} p_i \chi_i \right)^2 \tag{3}$$

which is the variance in NTD.

It has been demonstrated that this analysis enables recognition of the complex dynamic system under study entering the critical stage [1,9,10]. This occurs when the variance κ_1 converges to 0.07. Originally this for the approach to criticality was theoretically derived for the seismic electric signals (SES) [8], which are transient low frequency ($\leq$1 Hz) electric signals that have been repeatedly observed before earthquakes.

It was shown that k_1 satisfies the statistical properties of an OP [8], when it is used to characterize the phase transitions of complex systems in critical states and nonequilibrium systems. Varotsos et al. [1] have identified the features of the probability distribution of the OP for seismicity approaching the criticality, investigating, in particular, the behavior of the OP before the occurrence of the major earthquakes in California.

To our knowledge, no similar investigations have been performed on synthetic seismicity, and the study carried out in this paper tries to fill such a gap. We, therefore, analyze the fluctuations of an experimental array in which the relative movement of two rough surfaces (sandpapers) simulates the interaction between tectonic plates. Analogously to the real seismicity, an OP for the synthetic seismicity in NTD was defined and its PDF analyzed.

3. Experimental setup

Our experiments were aimed to simulate the interaction between two rough fault planes. Thus, we build up a frictional system, namely in the stick–slip process of a spring–slider setup, subjected to a mechanical forcing. The spring–slider system is considered as a proxy of geological faults under tectonic stress. Chelidze et al. [18,19] designed a similar stick–slip experiment in which the acoustic emissions during stick–slip is a proxy of seismic activity on the active fault.

Our experimental array, designed by Vargas et al. [20,21], which is characterized by two sandpapers in relative movement mimicking the relative motion of two tectonic plates. This experimental device, based on models proposed by Burridge and Knopoff [22] and Feder and Feder [23], is schematically shown in Fig. 1. The system consists of an aluminum block (a) of 0.1 m length, 0.1 m width, 0.025 m height, and 0.5 kg mass, which slides over a frictional surface with asperities (c), consisting of an aluminum track of dimensions 0.7 m length, 0.22 m width and 0.003 m height. The inferior surface of the block (a) and the aluminum track (c) are coated with sandpapers with different roughness degrees. Below the aluminum track, a low friction suspension system consisting of two glass plates was settled. The superior glass plate has a thickness of 0.009 m and rests on a set of steel spheres (e), with diameter of 0.004 m, which can roll over a second glass plate of 0.012 m thickness (d). All the suspension system is placed over a metallic frame to maintain it in a leveled position. The object (b) is a charge cell (Omega LCL), which works as a bumper against the metallic frame and allows us to record the force exerted by the inferior plate over the cell when the elastic rope (g) is kept in tension. The rope is a fishing string with a diameter of 5×10^{-4} m and a charge