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Trapping on modular scale-free and small-world networks with multiple hubs

Jin Chen [a], Meifeng Dai [b,*], Zhixiong Wen [a], Lifeng Xi [c]

[a] School of Mathematics and Statistics, Huazhong University of Science & Technology, Wuhan, 430074, PR China
[b] Nonlinear Scientific Research Center, Faculty of Science, Jiangsu University, Zhejiang, Jiangsu, 212013, PR China
[c] Institute of Mathematics, Zhejiang Wanli University, Ningbo, 315100, PR China

HIGHLIGHTS

- Generalize random walks on single-hub networks to multiple-hub networks.
- The scalings between the MFPT and average shortest path on multiple-hub networks are the same.
- MFPT on multiple-hub networks increase sublinearly with network order.

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ABSTRACT

In this paper we introduce a new kind of scale-free and small-world networks with multiple hubs based on classic networks and present the trapping problem on them. We show that in the multiple-hub networks, the growth of MFPT with the traps located on the hub or peripheral nodes displays quite a difference from the single-hub networks. Our conclusions are directed to possible applications in rumor or epidemic spreading phenomena.

1. Introduction

In the past two decades a great deal of attention has been paid to the research of complex networks, which are ubiquitous in nature and society such as the social networks, the World Wide Web [1], metabolic networks [2,3], and a network of routers connected by various physical connections [4,5]. Despite their diversity, most networks appearing in the real world obey some organizing principles, which lead to systematic and measurable deviations basing on the random graph theory originating from Erdős and Rényi [6,7]. In particular, three properties of real networks have aroused considerable scientific and technological interest. First, many real networks own modular (hierarchical) topology, this kind of network can be divided into several repeated groups, usually the nodes in the same group are highly interconnected with each other, but there are a few links between two nodes in different groups. For example, in society there exist groups of friends or coworkers [8], while in the World Wide Web there exist communities with shared interests [9]. Second, lots of measurements show that most networks display the scale-free property [10], which means that the probability that a randomly selected node with exactly k links decays as a power law, following $P(k) \sim k^{-\gamma}$, where γ is the degree exponent. Finally, lots of networks have the

* Corresponding author. Tel.: +86 13815158555.
E-mail addresses: daimf0225@163.com, daimf@mail.ujs.edu.cn (M. Dai).

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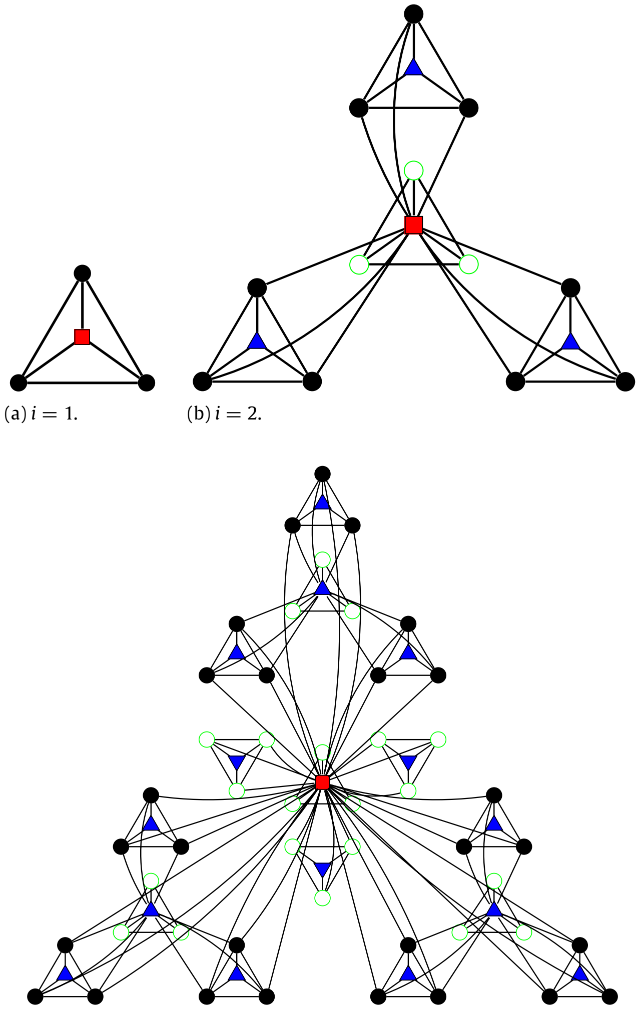

(a) $i = 1$. (b) $i = 2$.

(c) $i = 3$.

Fig. 1. (Color online) The iterative generating process of a modular network for the case of $N = 4$ and $n_1 = 1$. The solid circles, hollow circles, solid squares, and solid triangles are the peripheral nodes, the locally peripheral nodes, the hub nodes, and the locally hub nodes, respectively.

small-world behavior [11], namely, the typical distance between two uniform randomly chosen nodes grows proportionally to the logarithm of the number of nodes, simultaneously they display high average clustering coefficient.

An open problem about complex networks is to figure out how the topological structure affects the dynamical processes operating on networks [12]. Of all the dynamical processes, the trapping problem is one of simple random walks [13,14] which has caused great concern within the physics community [15–17]. For example, in Ref. [18], they consider the trapping problem in single-hub networks with modular organization and scale-free structure. Nowadays, the study of random walks or information transfer on multiple-hub networks has become more and more popular, such as communication networks [19] and hierarchical peer-to-peer networks [20]. Hence we will focus on the research of construction and dynamic properties for a kind of multiple-hub network.

In the first half of this paper, we give the construction of modular complex networks which are extensions of networks defined in Ref. [18] in some sense. Then we show that the described networks are scale-free and small-world. In the second half, we concentrate on the trapping problem with the traps located on the hub and peripheral nodes. When the hub is multiple, under some reasonable assumption, FMPT will increase linearly with the logarithm of the order of the network, which is very different from the result of the single-hub networks [18].

2. Modular scale-free and small-world networks

We start by constructing the networks with a modular structure in an iterative way. Prior to this, we introduce some notations for the convenience of the following statement, write G_i for the network model of the i-th ($i \geq 1$) generation, assume n_1 and N are positive integers satisfying $\max\{3, 2n_1 + 1, n_1 + 2\} \leq N$, called assumption A. Our starting point ($i = 1$) is a small cluster of N completely linked nodes (see Fig. 1(a)), which contains n_1 central nodes and $(N - n_1)$ external