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The predictive power of singular value decomposition entropy for stock market dynamics☆

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HIGHLIGHTS

- Correlation matrices of financial stocks.
- Singular value decomposition entropy.
- The entropy Granger-causes the overall dynamics of the stock market.

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ABSTRACT

We use a correlation-based approach to analyze financial data from the US stock market, both daily and monthly observations from the Dow Jones. We compute the entropy based on the singular value decomposition of the correlation matrix for the components of the Dow Jones Industrial Index. Based on a moving window, we derive time varying measures of entropy for both daily and monthly data. We find that the entropy has a predictive ability with respect to stock market dynamics as indicated by the Granger causality tests.

1. Introduction

The last financial and economic crisis has reignited the interest for a better understanding of the financial markets as well as for developing better tools to forecast their dynamics. Some possible new advances may come from the use of correlation matrices and their analysis.

The use of correlation matrices and of various methods to synthesize the resulting information can be traced back to Mantegna (1999) [1] who proposed the filtering of information with the help of minimum spanning trees. Further developments were made by Tumminello et al. (2005) [2], who implemented the planar maximally filtered graph which was shown to carry more information than the minimum spanning tree. A thorough study on the application of the planar maximally filtered graph was done by Tumminello et al. (2007) [3] on 300 stocks from the US market using computed different topological properties along time and different time frequencies.

More recently, given the huge impact of the financial crisis, many studies were done on the possibility of using correlation matrices to detect changes in the stock markets as well as the possibility of future crises. An early study is due to Onnela et al. (2003) [4] who studied the dynamics in time of an asset tree for which they proposed a length measure, finding that the length shrinks during critical events, like Black Monday.

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Another contribution has been made by Kenett et al. (2011) [5]. Using S&P 500 data with a sample between 1999 and 2010, they designed a so called index cohesive force based on the correlations between stocks. They showed that the index cohesive force, which can characterize the system as a whole, can be used to analyze the state of the market and the probability of a market crash.

More recent studies focused in analyzing the relationships between different world stock market indices. Song et al. (2011) [6] performed an analysis of the correlations between world stock market indices at a daily frequency. Two kinds of dynamics were identified, the *slow dynamics*, which could be interpreted as depicting the tendency toward more globalization, and the *fast dynamics*, which could be linked with critical events.

Kumar and Deo (2012) [7] realized a comparative analysis of the relationships between the world market indices using different techniques, like multifractal analysis, random matrix theory and networks. Their main finding was that the network approach provides the most useful information.

Kenett et al. (2012) [8] extended the use of the earlier proposed index cohesive force (see above) for the case of the world financial markets. In their study, they used both inter and intra correlations, the market index cohesive force and meta-correlations through which the dynamics of world capital markets were analyzed. Among the most important findings was that there are different patterns for developed Western markets and emerging Asian markets.

This paper aims at computing the entropy based on the singular value decomposition of the correlation matrices between the components of the main US stock market index, the Dow Jones Industrial Average. We use both daily and monthly data in order to check whether the results are sensitive to the frequency chosen. We also run Granger causality tests between the resulting time varying entropy indices and the Dow Jones Industrial Average.

The paper is organized as follows. The following section is dedicated to a discussion of the methods used in the paper. In the third section, we construct the entropy measures and use it to analyze the dynamics of the stock market. The last section is dedicated to a discussion of results and suggestions for future research.

2. Methodology

This section is dedicated to a short presentation of the methodological techniques used throughout the paper.

2.1. Correlation matrices of stocks

Probably the standard way to construct correlation matrix from financial time series is to use Pearson correlations between the different stocks, see Kenett et al. (2011) [5] for a comparative review of the approaches used in the literature. New developments include the use of a significance threshold, or the use of conditional correlations. A correlation matrix R can be constructed from correlations using the following formula:

$$R_{i,j} = \frac{\left\langle (y_i - \langle y_i \rangle) - \left(y_j - \langle y_j \rangle\right) \right\rangle}{\sigma_i \sigma_j} \tag{1}$$

where $\langle\rangle$ stands for the mean of the returns of a stock, while σ_i and σ_j are the standard deviations of the returns of stocks y_i and y_j, respectively. The return of a stock i is simply given by the logarithmic difference of the value of a stock, namely by:

$$y_{i,t} = \log(S_{i,t}) - \log(S_{i,t-1}) \tag{2}$$

where $y_{i,t}$ is return of the stock i while $S_{i,t}$ is the value of the stock i at moment t.

2.2. Singular value decomposition

Any matrix $A(mxn)$ can be decomposed using the singular value decomposition as:

$$A = USV^T, \tag{3}$$

with U an mxk matrix and V an nxk matrix. S is a diagonal matrix defined by:

$$S = \text{diag}\,(\lambda_1, \ldots, \lambda_k)\,, \tag{4}$$

where $k = \min(m, n)$. The values of the matrix S are both nonnegative and ordered from the biggest to the lowest elements.

2.3. Entropy

We can construct a complexity measure of the network, an entropy measure, using the singular values λ_k, following Sabatini (2010) [9]. The original idea of entropy dates back to Shannon (1948) [10].