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Dynamic fluctuation model of complex networks with weight scaling behavior and its application to airport networks

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HIGHLIGHTS

- We propose a dynamic fluctuation model of complex networks.
- The fluctuations in growth rates of weight are exponential.
- The evolution of weight governs the topological growth of the networks.
- This model quantitatively describes and reproduces the real airport networks.

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ABSTRACT

Airport networks are a kind of complex systems that display significant fluctuations. To describe this kind of phenomena a dynamical fluctuation model is proposed. In this model the fluctuations in growth rates of weight are exponential. There are two adjustable parameters: the exponent β and the creation probability P_d of a new node. β characterizes the tendency to retain an "optimal" allocation of weights (size of components) for a given kind of networks. P_d reflects the interactive effect between the weight fluctuation and variation of nodes and links (topological structure) for each real network. For airport networks the exponent β is 0.4 and the creation probability P_d is equal to 0.018 (China), 0.025 (US), 0.03 (Brazil) and 0.04 (Europe). This model quantitatively describes and reproduces these real airport networks.

1. Introduction

The network theory has undergone a remarkable development over the last decade and has become an important tool to better understand social, technological, and biological real-world systems since 1998 [1–6]. Many complex systems are not static but show significant fluctuations at various time scales [7]. Moreover, although there exist stable statistic regularities at the global level, many systems exhibit intense activity at the level of individual components, i.e., at the "microscopic" level [7]. In a previous study, Batty found that although the population Zipf plots display negligible changes in time, the same city can have very different ranks in the course of history [8]. Recently, Gautreau et al. provided a simple and obvious evidence for the presence of a microscopic dynamics [7]. They found that the total traffic of the US air transport system displayed strong fluctuations at the month scale, which, however, can be fitted by an exponential growth when the seasonal effects were averaged. They also found that the average degree of the US airport network rapidly varied with time. The average degree is approximately constant and is of order 15 at the global level. In order to describe the behaviors of the

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dynamic systems with strong fluctuations, they proposed a microdynamics model of stationary complex networks, which reproduced most empirical observation of the US airport network [7]. To accurately describe the properties of real systems, the fluctuation regularities should be established. Airport networks are typical dynamic systems with strong fluctuations. Taking into account the influence of economics, culture, tourism, climate, geography and geo-politics, the evolution of airport networks becomes very complex. Airport networks thus can be chosen as the representative in the investigation of the fluctuation regularities in dynamic systems with strong fluctuations.

The network theory has been used as an important tool in air transport systems. Some essential characteristics of airport networks have been explored [7,9–13]. The investigation of airport networks has taken three stages.

At the beginning stage, the network theory is used for describing air transport systems. Based on the observations of real air transport systems, physicists empirically study the topological structure of airport networks and found the characteristics of airport networks. These investigations are spread over wide areas. Many airport networks are established, such as the World-wide airport network [14,15], US airport network [7,9], European airport network [10], Chinese airport network [10–12], and Brazilian airport network [13] et al., which lead to the conclusion that the airport networks are small worlds, including a short average path length and a high degree of clustering.

At the second stage, some specific mechanisms and models are proposed. These models display the main features of some real airport networks. Taking into account the preferential attachment with the geo-political constraints, Guimerà et al. proposed a model that explains the fact that some airports with very small degree have a very high betweenness [16]. Some models [17,18], which consider the effect of the dynamics on strength of interaction between nodes, are designed for the evolution of weight-evolving networks. These models well depict the structural characteristics of corresponding airport networks. When a new node establishes, the model time increases a unit in these models. However, the mechanism of increasing airports in these models is not equivalent to that in real air transport systems. Therefore, these models could hardly describe the growth of the real air transport systems with time. Moreover, in these models, the proportion between nodes (airport) and edges (air routes) is fixed and the number of edges can increase and cannot reduce. Obviously, these assumptions are inconsistent with reality.

At the third stage, physicists investigate the characteristics of the airport network dynamics and try to quantitatively describe the real air transport systems. Recently, Gautreau et al. studied the features of the US airport network and observed a strong fluctuation at various time scales. They proposed a model of microdynamic fluctuating networks [7]. When this model is applied to airport networks, the stochastic fluctuation of weights takes place. This model reproduced the main empirical features of the US airport network. However, the standard deviation of the distribution of the weight's growth rate is fixed in this model. For the sake of simplicity, the model chooses the growth rates of weight as a random variable drawn from a distribution independent from time and from the weights. That is to say, no matter what the scale of the airport's traffic flow is, the average growth rate of weight is the same for each air route, which is obviously inconsistent with reality. Furthermore, the distribution of the logarithmic growth rate is Gaussian in this model but is exponential in reality.

In this paper we propose a dynamic fluctuation model of complex networks in which the fluctuations in growth rates of weight are exponential with the exponent β. This model was used for quantitative description and reproduction of real air transport systems, such as the US, Chinese, European and Brazilian airport networks.

2. Model

Many complex systems with strong fluctuations can be described with dynamic fluctuating networks. Airport networks are undirected networks with strong fluctuating weights. In airport networks, the node is the airport and the link is the air route. The degree k_i indicates the number of connected airports with a given airport i, the weight w_{ij} represents the number of traffic (passengers) per month at a given air route (i, j). The strength s_i is the sum of the weights at the node i, so s_i represents the total traffic handled by the airport i in an airport network. Usually, the topology characteristics of complex networks can be described by the average degree $\langle k\rangle = \frac{1}{N}\sum_i k_i$, average shortest-path length $d = \frac{1}{N(N-1)}\sum_{i\neq j} d_{ij}$, and clustering coefficient $C = \frac{1}{N}\sum_i C_i$, where N is the node number, d_{ij} is the distance between two nodes i and j, i.e. the number of edges along the shortest path connecting them, C_i is the ratio of the actual edge number (E_i) between its neighboring nodes to the possible edge number between them, i.e. $C_i = \frac{E_i}{k_i(k_i-1)/2}$.

Since the traffic is affected by many factors, the weight w_{ij} fluctuates. We assign the growth rate of weight by

$$\eta_{ij}(t) = \frac{w_{ij}(t) - w_{ij}(t-1)}{w_{ij}(t-1)} \tag{1}$$

and the logarithmic growth rate of weight by

$$r_{ij}(t) = \ln\left(\frac{w_{ij}(t)}{w_{ij}(t-1)}\right) = \ln(\eta_{ij}(t) + 1). \tag{2}$$

The random variable r_{ij} has a probability distribution which depends only upon $w_{ij}(t-1)$, and this dependence is independence of the air route (i, j).

The airport networks are somewhat similar to the company networks. Based on empirical data, Buldyrev et al. proposed that the growth rate of a company was affected by a tendency to retain an "optimal" size, and the distribution of the