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Impact of bounded noise and shortcuts on the spatiotemporal dynamics of neuronal networks

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HIGHLIGHTS

- Studying the effects of bounded noise on the dynamics of neuronal networks is urgent.
- Bounded noise with proper amplitude can result in temporal coherence in NW networks.
- Increasing noise amplitude will impair spatial synchronization among coupled neurons.
- An optimal amount of shortcuts can induce much more ordered spatiotemporal patterns.
- Shortcuts-induced much more ordered states are robust against Wiener process.

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ABSTRACT

The influences of bounded noise together with shortcuts on the spatiotemporal collective behaviors of temporal coherence and spatial synchronization are discussed in neuronal networks. Firstly, we focus on the case of regular neuronal networks. With the increase of noise amplitude, we find that the spatial synchronizability among coupled neurons is always impaired and coherence resonance, however, occurs at an appropriately tuned level of noise amplitude. Then, we introduce shortcuts to the regular neuronal networks to formulate small-world neuronal networks. The results indicate that the spatial synchronization and temporal coherence in the networks can be enhanced with the addition of shortcuts. Moreover, we verify that there exists an optimal amount of added shortcuts such that the small-world networks reach much more ordered spatiotemporal patterns, i.e., coupled neurons are nearly synchronized in space and most coherent in time. In addition, the shortcuts-induced much more ordered states are confirmed to be robust against changes of the intensity of the unit Wiener process.

1. Introduction

Noise, inevitable in neuronal systems, is intuitively disadvantageous to the spike dynamics for the case that it can destroy synchronization in coupled neurons [1]. Meanwhile, a lot of research including theoretical analysis, numerical simulations and experimental work has revealed the counterintuitive influences of noise in the literature. The most famous achievement demonstrating the constructive roles of noise is noise-induced coherence dynamics in neuronal systems [2–6]. For instance, Pikovsky and Kurths reported that Gaussian white noise can evoke coherence resonance in the FitzHugh–Nagumo system [2]; Hu and Zhou revealed the behavior of coherence resonance can also appear in a lattice of coupled non-identical neurons subject to independent external noise [4]; Baltanás further confirmed that stochastic resonance can be induced by Gaussian white noise in the Hindmarsh–Rose system [5]. Besides noise-induced coherence patterns, noise has manifested its beneficial roles by generating or assisting synchronous dynamics in neuronal systems

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[7–9]. For example, Zhou et al. verified that noise is able to induce phase synchronization in an ensemble of non-identical neurons [8]; He et al. illustrated that two non-coupled identical neurons can be synchronized by common noise [9].

It is a fact that a neuron in the vertebrate cortex may connect to more than ten thousands postsynaptic neurons via synapses, which in turn leads to neuronal networks with complex topology [10]. Networks with realistic feature, such as small-world property, have been experimentally verified to be good descriptions of interacted neurons [11–14]. Small-world architectures [15], presenting clear clustered structure and sparsely long-range random connectivity, are thus to be widespread in neuronal systems. Recently, the neuronal spike dynamics on small-world networks has been a topic of great relevance in theoretical neuroscience [16]. Much scientific research has been devoted to discussing the subtle effects of random long-range links or shortcuts on the dynamics of neuronal networks. On a small-world network of recurrently coupled excitable neurons, Riecke et al. found that persistent activity emerged at low density of shortcuts, and the system underwent a transition to failure as their density reached a critical value [17]. In neuronal networks with small-world topology, it was examined that noise-induced coherence resonance can be enhanced by introducing an intermediate fraction of shortcuts [18–22]. A comprehensive investigation performed by Perc et al. for excitable media also showed that coherence resonance can be enhanced by shortcuts [23–25]. More interestingly, the pacemaker driven stochastic resonance was described in noisy excitable small-world networks [26–29], where we can find that for a suitable fraction of rewired links the spreading of pacemaker activity is pronounced best. Moreover, on a modular neuronal network consisting of several small-world subnetworks, Yu et al. found that an optimal number of links between different subnetworks can warrant the emergence of pronounced stochastic resonance [30].

As mentioned above, much information, concerning the remarkable influences of shortcuts on noise-induced dynamics of neuronal networks, has been published. The important result that an intermediate fraction of shortcuts can optimize noise-induced dynamics has also impressed us. Nevertheless, the concerned noise in most of those cases is assumed to be of Gaussian behavior. Note that the range of Gaussian distribution is unbounded, namely, there exists a possibility of having very large values. This violates the fact that a real physical quantity is always bounded [31]. More importantly, experimental results have demonstrated that there could be non-Gaussian noise source in sensory systems like neurons of crayfish [32] and rat skin [33]. Bounded noise, a sinusoidal function with a constant average frequency and random phase modeled as a unit Wiener process, is a typical kind of non-Gaussian noise. In addition, the spectrum of bounded noise is mainly determined by the intensity of the unit Wiener process (σ). It is a narrow-band process when σ is small enough, and it approaches white noise when $\sigma \to +\infty$. Thus exploring the influences of bounded noise, a good candidate for non-Gaussian noise, on the spatiotemporal dynamics of neuronal systems is of great significance. Recently, some researchers have modeled external or intrinsic random disturbance as bounded noise in some nonlinear systems [31,34–37]. For instance, Liu et al. studied the effects of bounded noise on the chaotic behavior of Duffing oscillator under parametric excitation [35]; Yang et al. detected the twofold roles of bounded noise on complete synchronization in two non-coupled chaotic oscillators [36]; Yung et al. investigated the phenomenon of stochastic resonance in the FitzHugh–Nagumo neuron under combined bounded noise and weak harmonic excitation [37].

Motivated by the above findings, we wonder how bounded noise affects the spatiotemporal dynamics of small-world neuronal networks and then how shortcuts influence the noise-induced behaviors. To the best of our knowledge, there is no attention devoted to these issues until now. In the present work, we mainly explore the diverse roles of bounded noise and shortcuts on shaping the coherence and synchronous dynamics of small-world neuronal networks. The rest of this paper is organized as follows. Section 2 presents the description of bounded noise and the model of considered neuronal networks. In Section 3, main results concerning the effects of bounded noise and shortcuts on neuronal dynamics are discussed. Finally, a conclusion is made in Section 4.

2. The mathematical model: small-world neuronal networks driven by bounded noise

Before introducing the model of small-world neuronal networks, we first give a description of bounded noise. Its expression is as follows [31,34–37]

$$\xi(t) = \sin\left[\Omega t + \sigma B(t) + \Gamma\right] \tag{1}$$

where Ω is an average frequency, $B(t)$ is a unit Wiener process, and σ represents the intensity of $B(t)$, Γ is a random variable uniformly distributed in $[0, 2\pi]$. $\xi(t)$ is a stationary stochastic process in wide sense with zero mean. Its autocorrelation function is

$$C_\xi(\tau) = \frac{1}{2}\exp\left(-\frac{\sigma^2\tau}{2}\right)\cos(\Omega\tau), \tag{2}$$

and its spectral density can be expressed as

$$S_\xi(\omega) = \frac{\sigma^2}{2\pi}\left(\frac{1}{4(\omega-\Omega)^2+\sigma^4} + \frac{1}{4(\omega+\Omega)^2+\sigma^4}\right). \tag{3}$$

The variance of $\xi(t)$ is $C_\xi(0) = 1/2$. Fig. 1 shows the spectral density of $\xi(t)$. It is seen that the position of the spectral density peak strongly depends on Ω and the bandwidth of $\xi(t)$ is mainly determined by σ. It is a narrow-band process when σ is small enough, while it approaches white noise when $\sigma \to +\infty$.