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The entanglement temperature of the generalized quantum walk

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HIGHLIGHTS

- The asymptotic equilibrium between position and chirality is studied.
- We propose a temperature function in order to characterize this equilibrium.
- We show that the initial condition determines the equilibrium temperature.
- We calculate numerically the temperature as a function of the initial condition.

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ABSTRACT

We study the asymptotic equilibrium between the degrees of freedom of position and chirality in a generalized quantum walk on the line. For this system, we propose a temperature function in order to characterize this equilibrium. We show that the initial condition determines the equilibrium temperature. We calculate numerically the temperature as a function of the initial condition.

1. Introduction

Recently the asymptotic behavior of the quantum walk on the line (QW) has been investigated [1–3], focusing on the global chirality distribution (GCD) independently of position. This distribution has a long-time limit that depends on the initial conditions. Therefore, although the dynamical evolution of the QW is unitary, the evolution of its GCD has an asymptotic limit characteristic of a diffusive behavior. This result is further surprising if we compare the case studied in those papers with the case of the QW on finite graphs [4] where it is shown that there is no convergence to any stationary distribution. The stationary long-time limit behavior is usually associated with a Markovian process and not with a unitary process. Concepts such as thermodynamic equilibrium seem impossible to coordinate with the idea of a system that follows a unitary evolution and thus cannot reach a final equilibrium state at $t \to \infty$. In this context Ref. [2] shows that it is possible to introduce the concept of temperature for an isolated quantum system that evolves in a composite Hilbert space. Using the QW as a model Ref. [2] defined a thermodynamic equilibrium between the QW degrees of freedom of position and chirality and further introduced a temperature concept for this unitary closed system. Additionally in this reference the transient behavior towards thermodynamic equilibrium is described by a master equation with a time-dependent population rate.

On the other hand, several authors [5–16] have studied the QW subjected to different types of coin operators and/or sources of decoherence to analyze and verify the principles of quantum theory as well as the passage from the quantum to the classical world [5]. In Ref. [6] it was shown that the phases in the coin operator and the initial state of the coin can be used to control the evolution of the QW. The presence of decoherence in the QW has been studied as one possible route to

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classical behavior [7,8,16]. The appearance of a small decoherence can be used to enhance some properties of the QW in the development of quantum algorithms [7]. The QW subjected to multiple independent coins was analyzed in Ref. [17], showing that this type of excitation can lead to a diffusive spreading instead of the ballistic one. The QW with a position dependent phase is studied in Refs. [9,10] where dynamical localization was found. A coin with explicit time-dependence is introduced for the first time in Ref. [11], also finding dynamical localization and quasi-periodic dynamics. In Ref. [12] a non-linear dependence on the position–chirality probabilities is introduced in the evolution, finding a variety of dynamical behaviors, including ballistic motion and dynamical localization. The QW was also generalized [13] introducing two coin operators arranged in quasi-periodic sequences following a Fibonacci prescription, leading to a sub-ballistic wave function spreading, as shown by the power-law tail of the standard deviation ($\sigma(t) \sim t^c$ with $0.5 < c < 1$). Finally, the QW subjected to measurements [14] and decoherence with a Lévy waiting-time distribution [15] were studied. In both cases it was found that the system has a sub-ballistic spreading.

Here we shall focus on the generalized QW developed in Ref. [18] that generalizes the discrete quantum walk on the line using a time-dependent unitary coin operator. In that work an analytical relation between the coin operator and the long-time behavior of the standard deviation is found. Selecting the coin time sequence allows us to obtain a variety of predetermined asymptotic wave-function spreadings: ballistic, sub-ballistic, diffusive, sub-diffusive, and localized. The appearance of this variety of spreadings shows the existence of a wealth of quantum behaviors that are not restricted to be diffusive (decoherent) or ballistic (coherent). In Refs. [14,15] some analytical results were obtained about sub-ballistic behavior in a stochastic frame, but they cannot be extended to deterministic cases such as in Ref. [13]. Then it remains to clarify how some deterministic sequences of the coin operator lead to a behavior that is neither diffusive nor ballistic.

In this paper we use a QW model with a generalized coin [18], that allows an analytical treatment to study the diffusion coefficient; however to extend the concept of temperature developed in Ref. [2] we must work both analytically and numerically.

The paper is organized as follows. In the next section we develop the QW model with a time depended coin, in the third section the global chirality distribution and the density matrix are presented, while in the fourth section entanglement entropy and temperature of the generalized QW are developed. In the last section we draw the conclusions.

2. QW on the line with discrete time dependent coin

The standard QW corresponds to a one-dimensional evolution of a quantum system (the walker) in a direction which depends on an additional degree of freedom, the chirality, with two possible states: "left" $|L\rangle$ or "right" $|R\rangle$. The global Hilbert space of the system is the tensor product $H_s \otimes H_c$ where H_s is the Hilbert space associated to the motion on the line and H_c is the chirality Hilbert space. Let us call T_- (T_+) the operators in H_s that move the walker one site to the left (right), and $|L\rangle\langle L|$ and $|R\rangle\langle R|$ the chirality projector operators in H_c. We consider the unitary transformations

$$U(\theta) = \{T_- \otimes |L\rangle\langle L| + T_+ \otimes |R\rangle\langle R|\} \circ \{I \otimes K(\theta)\}, \tag{1}$$

where $K(\theta) = \sigma_z e^{-i\theta\sigma_y}$, I is the identity operator in H_s, and σ_y and σ_z are Pauli matrices acting in H_c. The unitary operator $U(\theta)$ evolves the state in one time step τ as $|\Psi(t+\tau)\rangle = U(\theta)|\Psi(t)\rangle$. The wave vector can be expressed as the spinor

$$|\Psi(t)\rangle = \sum_{k=-\infty}^{\infty} \begin{bmatrix} a_k(t) \\ b_k(t) \end{bmatrix} |k\rangle, \tag{2}$$

where the upper (lower) component is associated to the left (right) chirality. The unitary evolution implied by Eq. (1) can be written as the map

$$a_k(t+\tau) = a_{k+1}(t)\cos\theta + b_{k+1}(t)\sin\theta \tag{3}$$

$$b_k(t+\tau) = a_{k-1}(t)\sin\theta - b_{k-1}(t)\cos\theta. \tag{4}$$

The standard deviation is defined as

$$\sigma \equiv \sqrt{M_2 - {M_1}^2}, \tag{5}$$

where

$$M_2 = \sum k^2 P_k, \tag{6}$$

$$M_1 = \sum k P_k, \tag{7}$$

and

$$P_k \equiv \sqrt{a_k^2 + b_k^2}. \tag{8}$$

The evolution of the standard deviation is a distinctive feature of the quantum walk. It is well known that the standard deviation spreads over the line linearly in time $\sigma(t) \sim t$, while its classical analog spreads out as the square root of time $\sigma(t) \sim t^{1/2}$, i.e. diffusively.

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