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# Mechanical properties of microcantilevers: Influence of the anticlastic effect

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ABSTRACT

Measurements of Young's modulus of microstructures are frequently based on dynamic tests on microbeams. The aim of this work is evaluating if the accuracy of these measurements is affected significantly by the anticlastic effect. A nonlinear model of cantilever's dynamic behavior is thus developed and applied to some characteristic cases. The obtained results show that, even if the introduced nonlinearity is small enough to allow a modal approach to be still applied, the anticlastic effect has a meaningful influence on measurement accuracies as it is evidenced by the dependence of the resonant frequency on vibration amplitudes. The proposed treatise permits determining the appropriate range of excitation amplitudes to be used during the experiments and consequently to reduce appreciably the intervals of uncertainty of the measurements.

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## 1. Introduction

Microcantilevers are widely used nowadays as measurement devices in a broad range of applications: scanning tunneling and atomic force microscopes, micro and nano tribology studies, biology (down to the level of single molecule assay), etc. [1–4].

Material properties of microstructures are also frequently assessed by means of tests on microbeams. Static tests are sometimes performed to correlate the load-to-displacement behavior with the elastic modulus of the material [5,6]. More often, dynamic tests are performed to assess the first bending mode frequency of the studied structure with the aim of evaluating the elastic modulus of the material [7–12]. Since frequency measurements are generally easy to be implemented, the employment of dynamic tests reduces the complexity of the experimental set-up while accuracy is often improved up to the level of a few percent. If the effects of other error sources (geometry, air damping, residual stresses, etc.) are also considered, the intervals of uncertainty in the evaluation of Young's modulus reported in the literature are generally of the order of 10%.

In [13] it has recently been proven that, in the case of slender beams loaded statically by a pure couple, due to the so called anticlastic effect, the flexural behavior of the structure can be significantly affected not only by its geometrical characteristics but also by the entity of the deflections. A slightly deflected microbeam could thus exhibit a different flexural stiffness with respect to the same structure undergoing higher loads. It could be reasonable to assume that the anticlastic phenomenon affects also the dynamic behavior of a cantilever beam. In this case, however, the boundary conditions of the approach given in [13] are not respected, since during each cycle of vibrations the bending moment is not constant but varies along the beam and in time. In the dynamic tests described in the literature, this effect is not taken into account. Only in [12] the influence of the anticlastic curvature is considered, but a static approach is applied. The aim of this work is evaluating the influence of the anticlastic effect on the accuracy of the measurements of the material properties of microstructures.

## 2. Semi-analytical model

The first modal shape of flexural vibrations of a cantilever beam depicted in Fig. 1 can be described in normalized form as [14]:

$$q_1(\zeta)=\left(\frac{1}{N_2}\right)\{\sin(\beta_1\zeta)-\sinh(\beta_1\zeta)-N_1[\cos(\beta_1\zeta)-\cosh(\beta_1\zeta)]\} \quad (1)$$

where $\zeta = z/L$, with $z$ being the longitudinal coordinate along the beam of length $L$, while $\beta_1$, in the case of a clamped-free beam, is $\beta_1 = 1.875104$, and:

$$N_1 = \frac{\sin\beta_1 + \sinh\beta_1}{\cos\beta_1 + \cosh\beta_1}$$
$$N_2 = (\sin\beta_1 - \sinh\beta_1) - N_1(\cos\beta_1 - \cosh\beta_1) \quad (2)$$

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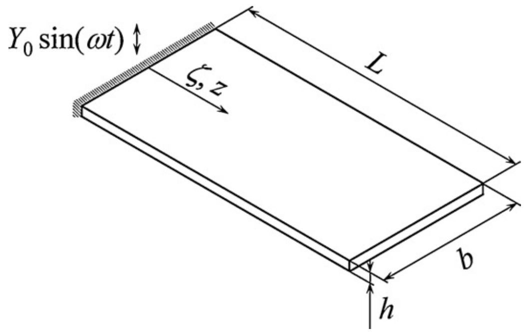


**Fig. 1.** Microcantilever structure.

The actual shape of the cantilever is thus:

$$u_1(\zeta, t) = \eta_1(t) q_1(\zeta) \tag{3}$$

where $\eta_1(t)$ is the modal coordinate of the first mode. Since in the considered case the eigenfunction expressed by Eq. (1) is normalized in such a way that the maximum value of the displacement is equal to unity, $\eta_1(t) = \bar{y}(t)$ i.e. $\eta_1(t)$ is equal to the amplitude of the displacement of the microbeam.

To take into account the anticlastic effect, the characteristic parameter $\alpha b$ defined by Angeli et al. [13] is:

$$\alpha b = \sqrt[4]{3(1-\nu^2)} \frac{b}{\sqrt{\bar{R}h}} \tag{4}$$

where, referring to Fig. 1, $b$ is the width of the cantilever, $h$ is its thickness, $\bar{R}$ is the curvature radius of the cantilever in the deformed position and $\nu$ is the Poisson's ratio of the beam material.

Since the first modal beam curvature $1/R$ is the second derivative of Eq. (1) with respect to $\zeta$:

$$\frac{1}{R(\zeta)} = \left(\frac{\beta_1^2}{L^2 N_2}\right) \{-\sin(\beta_1\zeta) - \sinh(\beta_1\zeta) - N_1[-\cos(\beta_1\zeta) - \cosh(\beta_1\zeta)]\} \tag{5}$$

obviously, according to Eq. (3), the beam curvature at a certain instant of time will be:

$$\frac{1}{\bar{R}(\zeta, t)} = \frac{\eta_1(t)}{R(\zeta)} \tag{6}$$

It is hence possible to adopt the correction factor $\Phi$ for the flexural stiffness of the beam as defined in [13]:

$$\phi = \frac{1}{1-\nu^2} - \frac{2\nu^2}{\alpha b(1-\nu^2)} F^*(\alpha b) + \frac{\nu^2}{2\alpha b(1-\nu^2)} f^*(\alpha b) \tag{7}$$

with

$$\begin{aligned} F^*(\alpha b) &= (B_1^* + B_2^*)\sinh\frac{\alpha b}{2}\cos\frac{\alpha b}{2} - (B_1^* - B_2^*)\cosh\frac{\alpha b}{2}\sin\frac{\alpha b}{2} \\ f^*(\alpha b) &= 2(B_1^{*^2} + B_2^{*^2})(\sinh\alpha b + \sin\alpha b) + (B_1^{*^2} - B_2^{*^2} + 2B_1^*B_2^*)\cosh\alpha b\sin\alpha b + (B_1^{*^2} - B_2^{*^2} - 2B_1^*B_2^*)\sinh\alpha b\cos\alpha b + 2(B_1^{*^2} - B_2^{*^2})\alpha b \end{aligned} \tag{8}$$

and

$$B_1^* = \frac{B_1}{\nu/\sqrt{3(1-\nu^2)}} \quad B_2^* = \frac{B_2}{\nu/\sqrt{3(1-\nu^2)}} \tag{9}$$

$$\begin{aligned} B_1 &= \frac{\nu}{\sqrt{3(1-\nu^2)}} \frac{\sinh(\alpha b/2)\cos(\alpha b/2) - \cosh(\alpha b/2)\sin(\alpha b/2)}{\sinh\alpha b + \sin\alpha b} \\ B_2 &= \frac{\nu}{\sqrt{3(1-\nu^2)}} \frac{\sinh(\alpha b/2)\cos(\alpha b/2) + \cosh(\alpha b/2)\sin(\alpha b/2)}{\sinh\alpha b + \sin\alpha b} \end{aligned} \tag{10}$$

The theory given in Angeli et al. [13] is based on the assumption that the load applied to the beam induces a constant curvature along its length. In the case considered in this work (frequency response of a microcantilever) the curvature varies continuously along the beam. It seems, therefore, reasonable to extend the same approach, thus obtaining a stiffness correction factor $\Phi$ that depends on the position $\zeta$ along the beam and varies also during each oscillation cycle, i.e. $\Phi = \Phi(\zeta, t)$. What is more, making the hypothesis that the system is slightly nonlinear, i.e. that the correction introduced by taking into account the anticlastic effect influences only slightly the dynamic response of the microbeam, it seems reasonable that a modal approach can still be used. If the usual methodology is applied [14], the "instantaneous" modal stiffness $\bar{K}_1$ of the microbeam can be evaluated as:

$$\bar{K}_1 = \int_0^1 \frac{bh^3 E\phi}{12(R(\zeta))^2} L d\zeta \tag{11}$$

where $E$ is Young's modulus of the beam material.

It must be noted that, with respect to the linear case, the usual expression of the modal stiffness is modified by introducing the correction factor $\Phi$ that is integrated along the whole length of the beam. Moreover, due to the mentioned small nonlinearity, the modal stiffness varies during the oscillation cycle, i.e. $\bar{K}_1 = \bar{K}_1(t)$, and therefore it has been indicated as "instantaneous".

The modal stiffness of the linear system $K_1$ is:

$$K_1 = \int_0^1 \frac{bh^3 E}{12(R(\zeta))^2} L d\zeta \tag{12}$$

Considering Eqs. (11) and (12) and bearing in mind that the limit values of the correction factor $\Phi$ are, respectively, 1 and $1/(1-\nu^2)$ [13], it follows that:

$$K_1 \le \bar{K}_1 \le \frac{K_1}{1-\nu^2} \tag{13}$$

In fact, depending on the oscillation amplitude, the "instantaneous" modal stiffness $\bar{K}_1$ could vary slightly between that of a beam-like structure $K_1$ and that of a plate bent to a cylindrical surface $K_1/(1-\nu^2)$ [15]. These bound values of flexural stiffness refer, respectively, to a plane stress and a plane strain structural model.

The well known expression of modal mass can be used:

$$\bar{M}_1 = \int_0^1 \rho bh[q_1(\zeta)]^2 L d\zeta \approx 0.25\rho Lbh \tag{14}$$

with $\rho$ designating the density of the cantilever.

As it will be shown below, due to the slight variation of $\bar{K}_1$, the response of the system is close to that of a linear system. A resonant frequency, i.e. the frequency at which the normalized frequency response is maximal, can thus still be expressed as:

$$\bar{\omega}_1 = \sqrt{\frac{\bar{K}_{1eq}}{\bar{M}_1}} \tag{15}$$

where $\bar{K}_{1eq}$ is the "equivalent" modal stiffness of the system. Physically, the latter is a kind of time average of the value of $\bar{K}_1$ that cannot be determined analytically, but it has to be obtained numerically (or experimentally).

In the case of excitations due to the harmonic motion of the supporting structure $y_0(t) = Y_0 \sin(\omega t)$, the response of the cantilever is obtained by resorting to a reference frame fixed to the constraint (Fig. 2) [14]. In this case the modal force is given by:

$$\bar{F}_1(t) = a\omega^2 Y_0 \sin(\omega t) \tag{16}$$

where $a$, in the case of a beam with uniformly distributed mass, is:

$$a = \left(\rho\frac{hbL}{N_2\beta_1}\right)\{-\cos\beta_1 - \cosh\beta_1 - N_1[\sin\beta_1 - \sinh\beta_1] + 2\} \tag{17}$$

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