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Classical limit of quantum mechanics induced by continuous measurements

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HIGHLIGHTS

- We investigate the quantum–classical transition problem in the newtonian regime.
- We show that the Newtonian regime occurs when the system is strongly monitored.
- We show that the Liouvillian regime is mimicked, for the position observable for a weak monitoring.
- We studied the quartic oscillator and our the numerical simulations confirm the analytical results.

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ABSTRACT

We investigate the quantum–classical transition problem. The main issue addressed is how quantum mechanics can reproduce results provided by Newton's laws of motion. We show that the measurement process is critical to resolve this issue. In the limit of continuous monitoring with minimal intervention the classical limit is reached. The Classical Limit of Quantum Mechanic, in Newtonian sense, is determined by two parameters: the semiclassical time (τ_{sc}) and the time interval between measurements ($\Delta\tau_u$). If is $\Delta\tau_u$ small enough, comparing with the τ_{sc}, then the classical regime is achieved. The semiclassical time for Gaussian initial states coincides with the Ehrenfest time. We also show that the classical limit of an ensemble of Newtonian trajectories, the Liouville regime, is approximately obtained for the quartic oscillator model if the number of measurements in the time interval is large enough to destroy the revival and small enough to not reach the Newtonian regime. Namely, the Newtonian regime occurs when $\tau_{sc} \gg \Delta\tau_u$ and the Liouvillian regime is *mimicked*, for the position observable, if $\Delta\tau_u \in [\tau_{sc}, T_R]$, where T_R is the revival time.

1. Introduction

Historically, immediately after the birth of the Quantum Mechanics many physicists believed that Quantum Mechanics was a universal theory, i.e. applicable to all Physics problems. In fact this idea was in the core of the Bohr–Sommerfeld quantization rule. However, later, it became clear that this was not the case. A different approach for high energies was necessary, as well as in the macroscopic "World". In 1917, Einstein [1,2] presented a reformulation of the Bohr–Sommerfeld quantization rules of the old quantum theory. The paper also offered an insight on the limitations of the old quantum theory when applied to a mechanical system that is nonintegrable. However, it had and in fact still it has, a trend to believe that Newtonian Mechanics is a particular case of the Quantum Mechanics, hypothesis that still opened, although great advances have occurred, some examples can be found in Refs. [3–29]. In the core of this question, the main aspect is the "status" of

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the Quantum Mechanics while the fundamental theory, at least for low energies. This is the Classical Limit of Quantum Mechanics problem (CLQM).

In the earlier times, the CLQM was studied in terms of Ehrenfest theorem [15–17,13,14,9,25–28] which states that, under certain conditions, the centroid of a wave-packet state will follow a classical trajectory. In a collection of papers [5,3,4], Ballentine and collaborators stated that the Ehrenfest's theorem is not adequate to characterize the classical regime, in their own words "A quantum state may behave essentially classically, even when Ehrenfest's theorem does not apply, if it yields agreement with the results calculated from the Liouville equation for a classical ensemble". and "We have shown that Ehrenfest's theorem is neither necessary nor sufficient to characterize the classical regime in quantum theory". Those results were later confirmed by others [22,8–10,18,19,24]. Zurek and Paz [7,21] have argued that no quantum system is isolated from their environment, thus we must consider a bigger system that includes the environment. In opposition, Wiebe and Ballentine have considered a coarse grained measurement [6] and concluded that "for all practical purposes, the quantum theory of the chaotic tumbling motion of Hyperion will agree with the classical theory, even without taking into account the effect of the environment. Decoherence aids in reducing the quantum–classical differences, but it is not correct to assert that the environmental decoherence is the root cause of the appearance of the classical world". In a recent paper [19], Oliveira and coworkers have shown that, in fact, there is a combination of factors. For a non-linear oscillator, they observed that "the agreement between quantum and classical mechanics is achieved through the convergence of three factors: large actions, the interaction with the environment and experimental observation limitations. Large classical actions are important for classicality during the whole dynamics, but it is not a sufficient condition: diffusion plays an essential role due to the presence of the revival in the quantum case. Deviations between the quantum and classical dynamics are not detectable if we take the experimental limitations into account". The experimental resolution and its relation with CLQM have been studied by many authors, see Refs. [23,30,11,3,29,31,19,32]. The decision whether a system is classical or quantum depends on the experimental apparatus which is essentially classical.

Recently, Angelo [11,29] has returned with the Einstein's question "inevitable conception that Physics must ferment a realistic description of only one system." Indeed, "nature as a whole may be thought of as an individual system existing only one time, with no need for repetitions and not as an 'ensemble of systems" see Refs. [11,29] and references there in. He concludes that the classical behavior only exists as an approximated notion derived from low-resolution measurements, "A scenario of quasideterminism may then be defined, within which the motion is experimentally indistinguishable from the truly deterministic motion of Newtonian mechanics. Beyond this time scale, predictions for individual systems can be given only statistically and, in this case, it is shown that diffusive decoherence is indeed a necessary ingredient to establish the quantum–classical correspondence".

There are distinct notions of classicality [33]. In this contribution, our aim is to shed a light under the Newtonian Classical Limit of Quantum Mechanics issue and contrast it with Liouvillian Classical Limit of Quantum Mechanics [19]. The fundamental point is the action of the measurement apparatus on the system, in fact this is closely related with decoherence program [31,34,25–28]. We consider a situation where the environment action is negligible and thus the system is isolated or the environment acts like a phase reservoir [11,29,18]. The essential idea here is that if one tries to make continuous simultaneous measurement of position and momentum (CMPM), then the information about the quantum nature of the particle will be lost.

Our procedure is similar to Refs. [25–28], they consider a quantum system entangled with the environment or the detector, formally they treat the system as a Wiener process, and in some circumstances, the purity of the state is preserved [27]. They numerically recovered the classical trajectories of regular and chaotic systems, they argue that the measurement process can be modeled by a stochastic master equation. According to Bhattacharya and collaborators [27] there is no reason to believe that the particular measurement model would affect the results and "any measurement or interaction which produces a localization in the phase space should lead to the classical behavior", and our results corroborate with that supposition. In this contribution, we investigate analytically and numerically this procedure. We show that in a CMPM the Quantum dynamics is indistinguishable of the classical Newtonian dynamics and we also show that the classical limit of an ensemble of Newtonian trajectories, the Liouville regime, is *approximately* obtained for the quartic oscillator model if the number of measurements in the time interval is large enough to destroy the revival and small enough to not reach the Newtonian regime. The Newtonian regime is related with a higher precision measurement in CMPM, which leads to a strong localization and in this case, the dynamics is governed by the semiclassical approximation, see next section and Refs. [9,20,35,36], this result is model independent.

This paper is organized as follows: in Section 2 we present the method, and an alternative view of Ref. [9]. In Section 3 we define a semiclassical time in terms of scalar products, overlap is considered. Section 4 we analyze the Newtonian Classical Limit of Quantum Mechanics problem. We show in an analytical way that the Newtonian regime is a natural consequence of using the CMPM. In Section 5 we use the quartic oscillator as model. Section 6 contains conclusions.

2. The semiclassical expansion

We consider a general quantum density operator $\widehat{\rho}$. Its time evolution is given by

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\rho} = -[\widehat{\rho}, \widehat{H}] \quad (1)$$