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Derivation of constitutive data for flowing fluids from comparable data for quiescent fluids

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ABSTRACT

The principles of linear irreversible thermodynamics are used to show that near-equilibrium linear constitutive equations governing the diffuse fluxes through fluids of momentum, energy, and other extensive properties, and valid for the case of general unsteady flows, can be derived by inspection solely from knowledge of their steady-state quiescent-fluid counterparts. This includes predictions not only of the general forms of these flux-force constitutive equations but also of the values of the phenomenological coefficients appearing therein. To supplement these diffuse flux data so as to effect closure of the fundamental equations of hydrodynamics, constitutive knowledge is also required of the relation between the fluid's specific momentum density (momentum "velocity") $\hat{\mathbf{m}}$ appearing in the inertial term of the momentum equation, and its mass velocity $\mathbf{v}_m$ appearing in the continuity equation. Towards this end, and with $\mathbf{j}_v$ the diffuse flux of volume, a plausible argument is advanced favoring the view that $\hat{\mathbf{m}} = \mathbf{v}_m + \mathbf{j}_v$ over that of Euler's currently accepted (albeit implicit) hypothesis that $\hat{\mathbf{m}} = \mathbf{v}_m$, with the two possibilities currently a matter of contention.

1. Introduction

1.1. Primary goals

The principal purpose of this paper is to show, using the principles of linear irreversible thermodynamics (LIT) [1,2], that linear constitutive equations governing the diffuse fluxes [3] through fluids of momentum, energy, and other extensive properties under general unsteady-flow near-equilibrium circumstances can be derived in their entirety from knowledge of their steady-state, nonflow counterparts. The converse is, of course, always trivially possible, namely the ability to derive steady-nonflow formulas from their unsteady-flow counterparts by simply passing, mathematically, to the static fluid limit. However, the reverse possibility – the principal subject of this paper – could not have been anticipated. (The reader who is already moderately familiar with the contents of prior papers in this sequence might wish at this point to read the overview and summary of this paper's findings in Section 7.1.)

The surprising ability to effect this calculation is not limited solely to establishing the respective *forms* of the flux-force constitutive relations [1–3] in the unsteady-flow case for each of the extensive physical properties undergoing transport; rather, it also includes the ability to establish the values of the phenomenological coefficients appearing in these constitutive relations from knowledge of their steady-nonflow counterparts. These theoretical findings imply that data acquired from experiments, molecular dynamic simulations, and other such schemes performed on fluids that are at rest can be used to predict the outcome of experiments or simulations performed on these same fluids when they undergo flow, at least in near-equilibrium [1,2] circumstances.

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This ability can be traced in large measure to the fact that the fluid's rheological response to stimuli, normally regarded as manifested only during conditions of flow, also figures in the fluid's overall transport behavior even when the fluid is at rest. This attribute is already implicit in Burnett's [4,5] Boltzmann equation-based [5–7] constitutive expression for the stress tensor in dilute gases arising from temperature or pressure gradients. Indeed, in the nonisothermal case, this result, showing the existence of stress in a static fluid, was already given earlier by Maxwell [8,9] in his celebrated 1879 paper, "On the stress in rarefied gases arising from inequalities of temperature", wherein he identified a term proportional to $\eta\nabla\nabla T$ as contributing to the stress tensor (with the temperature T and η the viscosity—the latter being a rheological flow property not normally manifested during nonflow situations of the type under discussion).

Among other things, our work expands the science of rheology [10] to cover situations where the fluid, though at rest, undergoes volume transport. It is true that the transport of volume independently of shear has, since the time of Stokes [11], always figured in rheological science in connection with stress resulting from purely dilatational fluid motions (with the stress being quantified by the fluid's bulk or volume viscosity coefficient [12]). However, the circumstances described herein are uniquely different, since whereas the fluid undergoes mass motion in Stokes' case it does not do so in our static case.

This paper is a sequel to the previous paper in this series, [13]. Details contained therein provide background for the main thrust of what follows.

1.2. Secondary goals

A secondary goal of the present paper aims at promoting a plausible argument which, if accepted, would offer a rational resolution of the contentious debate [14] that has recently erupted in connection with Euler's [15] implied (although not explicitly stated by him) constitutive relation

$$\hat{\mathbf{m}} = \mathbf{v}_m \tag{1.1}$$

between the fluid's specific momentum density ("momentum velocity") $\hat{\mathbf{m}}$ and its mass velocity $\mathbf{v}_m$. The former velocity arises in connection with its presence in the inertial term in Cauchy's linear momentum equation [16,17]

$$\rho\frac{D\hat{\mathbf{m}}}{Dt} = -\nabla\cdot\mathbf{P} + \rho\hat{\mathbf{f}}, \tag{1.2}$$

with $\mathbf{P}$ the pressure tensor, ρ the mass density, and $\hat{\mathbf{f}}$ the specific body force. The mass velocity appears in Euler's continuity equation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}_m) = 0. \tag{1.3}$$

In the above,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v}_m\cdot\nabla \tag{1.4}$$

denotes the material derivative.

Belief in the correctness of the widely accepted Eq. (1.1) is implicit, although not explicit, in every fluid mechanics textbook written since its original adoption by Euler [15] in 1755. He, and most others since, view that relation as an identity, as witness the fact that both velocities, although very different in their respective physical connotations, are invariably assigned a common velocity symbol, typically $\mathbf{v}$, such that the momentum and continuity equations appear respectively in standard textbooks [3,16,17] as

$$\rho\frac{D\mathbf{v}}{Dt} = -\nabla\cdot\mathbf{P} + \rho\hat{\mathbf{f}}$$

and

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0.$$

Belief in the correctness of (1.1) for continua is based upon an analogy with Newton's momentum law governing the movement through empty space of a collection of discrete mass-point particles. That is, it represents an attempt to reconcile the notion of a so-called "fluid particle" [17] with that of a material particle.

As was apparently first recognized by Landau and Lifshitz [16, Footnote; p. 196], Eq. (1.1), rather than constituting an identity, is, in fact, a hypothesis; that is, it represents a constitutive relation between the symbols $\hat{\mathbf{m}}$ and $\mathbf{v}_m$. Landau and Lifshitz attempted to prove the hypothesis, arriving by seemingly plausible continuum-level arguments at the conclusion that Euler's intuitive proposal was indeed correct. The basic chronology of what ensued subsequently when the legitimacy of their proof was later questioned is set forth in [14]. On a broader philosophical basis, and independently of the preceding issues, Serrin [18] argued that continuum-mechanical laws such as (1.2) and, hence, by implication (1.1), cannot be proved by theoretical, first-principles arguments starting from particulate-mechanical laws; rather, at best, laws governing continua can only claim theoretical (rational) legitimacy on the basis of their being "plausible" when viewed in the light of an analogy