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Jamming in the weighted gradient networks

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ABSTRACT

We introduce a weighted gradient network on the unweighted substrate networks. The edge weight w_{ij} is defined as the power-law function of the scalar difference $|h_j - h_i|$ between the two connected nodes i and j, i.e., $w_{ij} \sim |h_j - h_i|^{\alpha}$. The jamming factor J of the whole network is defined as the average of the jamming degrees of all nodes. The jamming properties are studied for the substrate of random and scale-free networks. We find a crossover phenomenon in two networks. For $\alpha > \alpha_c$ (α_c denotes a critical parameter, estimated to be about 1.5 for the random network and about 2.0 for the scale-free network), the value of J increases with increasing average connectivity $\langle k \rangle$ of the network, while for $\alpha < \alpha_c$, the reverse occurs. In addition, the jamming effects of the two networks are compared, and we give the range of values of α and $\langle k \rangle$ that the scale-free networks have a higher level of congestion than the random networks.

1. Introduction

The relation between network structures and transport functions has become an important subject in the studies of complex networks [1–4] because this problem is encountered in a multitude of natural and human-made transport and communication systems. In order to explore how the network structure and routing rules affect transport dynamics on the networks, many typical transport models have been studied on complex networks [5–13].

Recently, the jamming problem of gradient-driven transport has attracted much interest in complex networks [14,15]. In these studies, a random scalar is assigned to every node in a substrate network, such as random networks [16], scale-free networks [17], etc., and then a gradient network is established by local gradients at nodes. In this gradient network, each node has only one outgoing link to a nearest neighbor or itself which has the highest scalar. The transported entities (such as information) can flow on the gradient network. The results in Ref. [14] showed that for the Erdos–Renyi random network, the degree of jamming tends to the maximal value with increasing network size N, while for the Barabási–Albert scale-free network, the congestion does not grow. Ref. [15] found that scale-free networks can allow efficient transport compared to random networks when the average degree $\langle k \rangle$ is larger than 10, and vice versa. These findings indicate that the jamming strongly depends on the average degree $\langle k \rangle$ of the complex networks.

In the model of previous studies, the transported entities flow from a node to its only one nearest neighbor node which has the highest scalar. However, in many real systems, the entities can be transported from a node to all its nearest neighbors that have higher scalar than itself, and the flux between the two nodes is determined by their scalar difference. According to the universal rule in physics, a larger scalar difference leads to a larger flux; hence, it is reasonable to consider the power-law expression for the weight of a link. In this paper, we construct a weighted directed gradient network on the substrate of

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unweighted complex networks. In this gradient network, a directed edge can be drawn from a node i to its nearest neighbor j which has higher scalar than i. The edge weight w_{ij} is determined by the scalar difference $(h_j - h_i)$ between the connected nodes i, j with a power-law function ($w_{ij} \sim |h_j - h_i|^\alpha$). We consider random and scale-free networks as the substrate networks and study the jamming properties for various values of the parameter α and the average connectivity $\langle k \rangle$. Our main finding is that in both two networks, there exists a critical parameter α_c above which ($\alpha > \alpha_c$) the value of J increases with $\langle k \rangle$, while below α_c ($\alpha < \alpha_c$) the opposite occurs. The numerical results showed that the critical value α_c is about 1.5 for the random network and $\alpha_c \approx 2.0$ for the scale-free network. In addition, the jamming properties of random and scale-free networks are compared. We find that there exists a more complicated crossover phenomenon, which is described by two parameters α and $\langle k \rangle$. If $\langle k \rangle < k_c$ (k_c denotes a critical connectivity), the level of the jamming in scale-free network is higher than that in random network for all α. If $\langle k \rangle > k_c$, there exists another critical value $\alpha_{c'}$ below which ($\alpha < \alpha_{c'}$) scale-free networks have a higher level of congestion than random networks, while the opposite occurs for $\alpha > \alpha_{c'}$. The critical value $k_c \approx 10$, which is consistent with the result in Ref. [15]. The value of $\alpha_{c'}$ is estimated to be about 8.

2. Weighted gradient network model

We constructed the weighted gradient networks as follows:

(i) An unweighted network of N nodes is given and called as the substrate network, such as, random and scale-free networks.
(ii) We assign a random scalar h_i (drawn from a uniform distribution in [0, 1]), which describe the "scalar" of the node.
(iii) For each node i, we examine all its neighboring nodes and a directed edge can be drawn from the node i to its near-neighbor j which has higher scalar than itself. If i has the highest scalar than all its neighbors, a self-linked loop is possible. If the node i is isolated, and a self-loop is also set up. The set of the outgoing neighbors of node i is denoted by $\phi_i (i \notin \phi_i)$. The set of the incoming neighbors of node i is denoted by $\varphi_i (i \notin \varphi_i)$.
(iv) The weight w_{ij} of edge from i to j can be defined as $w_{ij} = \frac{c(i)(h_j - h_i)^\alpha}{\sum_{j'} (h_{j'} - h_i)^\alpha}$, $\alpha \geq 0, j, j' \in \phi_i$, and $c(i)$ is the amount of information produced by the node i. It is obvious that $\sum_j w_{ij} = c(i)$. If the node i has a self-linked loop, the edge weight $w_{ii} = c(i)$. Here we set $c(i) = 1$ for all nodes.
(v) The gradient network can then be defined as the collection of all directed links. Apparently, there are no loops in the network except self-loops. In fact, in the limit $\alpha \to +\infty$, the outflow of information of the node is distributed to the only one outgoing neighbor which has the largest scalar among all the neighbors of this node. Then our model becomes consistent with the model in Ref. [14].

Let $M_{\text{out}}(i)$ and $M_{\text{in}}(i)$ represent the outflow and inflow of information of single node i, respectively. So that $M_{\text{out}}(i) = \sum_j w_{ij}, j \in \phi_i$ and $M_{\text{in}}(i) = \sum_j w_{ji}, j \in \varphi_i$. If the node i has not a self-linked loop $M_{\text{out}}(i) = c(i) = 1$, otherwise, $M_{\text{out}}(i) = 0$. It is obvious that the inflow of information $M_{\text{in}}(i)$ has different values for different nodes.

We denote $D(i)$ as the capacity of the node i and assume $D(i) = c(i)$. For the nodes having not a self-linked loop, if the total amount of inflow of the node $M_{\text{in}}(i) > D(i)$, there should remain $(M_{\text{in}}(i) - D(i))$ which cannot be processed by node i and so that $J(i) = M_{\text{in}}(i) - D(i)$, certainly, $J(i) = 0$ if $M_{\text{in}}(i) \leq D(i)$. For the nodes having a self-linked loop, the amount of flows $c(i)$ produced by the node i should be jammed. So that, $J(i) = M_{\text{in}}(i) - D(i) + c(i) = M_{\text{in}}(i)$ if $M_{\text{in}}(i) > D(i)$ and $J(i) = c(i)$ if $M_{\text{in}}(i) \leq D(i)$. The jamming factor of the network J can thus be defined as the average of the jamming degrees $J(i)$ of all nodes:

$$J = \left\langle \left\langle \frac{\sum_i J(i)}{N} \right\rangle_h \right\rangle_{\text{network}}$$

where $\langle \ldots \rangle_h$ and $\langle \ldots \rangle_{\text{network}}$ denote the statistical average over realizations of the set of scalar and the network configuration, respectively.

3. Jamming in the random networks

We considered random and scale-free networks as the substrate networks. The statistical average is performed over 1000 samples. The system size $N = 10\,000$. First we study the relation between the jamming factor J and the parameter α for the fixed average connectivity $\langle k \rangle$. Second we examine how the average connectivity $\langle k \rangle$ affects the jamming factor J for different values of α. Finally, the jamming properties of the two networks are compared with the same parameters.

For random network, we use the Erdos–Renyi random model [16], where each pair of nodes is linked with probability $p = \frac{\langle k \rangle}{N}$. Fig. 1(a) shows the jamming factor J_R for various values of α for $\langle k \rangle = 2$. It is shown that the jamming factor J_R increases with α, and in the limit $\alpha \to \infty$, J_R tend to a constant. In order to determine the exact value of the constant, the jamming factor $J_R(\alpha)$ is plotted as a function of $\frac{1}{\alpha}$ in Fig. 1(b). We find that $J_R(\alpha)$ is in direct proportion to $\frac{1}{\alpha}$ for $\alpha \geq 2$. So that we can assume jamming factor J_R and the parameter α have the following relations for $\alpha \geq 2$:

$$J_R(\alpha) = J_R(\infty) + \frac{\text{const}}{\alpha}. \tag{1}$$