Physica A 390 (2011) 3876–3887

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Effect of initial concentration and spatial heterogeneity of active agent distribution on opinion dynamics

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ARTICLE INFO

Article history:
Received 22 January 2011
Received in revised form 3 May 2011
Available online 12 June 2011

Keywords:
Sociophysics
Monte Carlo simulation
Finite-size scaling
Critical exponents

ABSTRACT

We analyze the effect of spatial heterogeneity in the initial spin distribution on spin dynamics in a three-state square lattice divided into spatial cells (districts). In the spirit of the statistical mechanics of social impact, we introduce a dominant influence rule (DIR), according to which, in a single update step, a chosen node adopts the state determined by the influence of its discussion group formed by the node itself and its neighbors within one or more coordination spheres. In contrast to models based on some form of majority rule (MR), a system governed by the DIR is easily trapped in a stable non-consensus state, if all nodes of the discussion group have the same weight of influence. To ensure that a consensus in the DIR system is necessarily reached, we need to put a stochastic process in the update rule. Further, the stochastic DIR model is used as a starting point for understanding the effect of spatial heterogeneity of active agent (non-zero spin) distribution on the exit probabilities. Initially, the positive and negative spins (active agents) are assigned to some nodes with non-uniform spatial distributions; while the rest of the nodes remain in the state with spin zero (uncommitted voters). By varying the relative means and skewness of the initial spin distributions, we observe critical behaviors of exit probabilities in finite size systems. The critical exponents are obtained by Monte Carlo simulations. The results of numerical simulations are discussed in the context of social dynamics.

1. Introduction

It is well known that many features of the collective behavior of large social systems are independent of the attributes of individuals and details of social interactions [1–4]. In this way, stochastic spin models on networks are widely used to study the general features of a wide variety of social systems with the ultimate aim to explain the occurrence at a global level of complex phenomena like the formation of hierarchies and consensus (see Refs. [1–14] and references therein). The use of spin models to simulate the opinion dynamics is based on the social observation that people tend to cooperate while exchanging their opinions, and these interactions cause an opinion shift towards consensus or compromise among some alternatives, reminiscent of the stable magnetic states of spin models on networks [15]. The aim of these simulations is not only academic, as the spin models permit to reproduce and study the general features of real social systems. Elections provide a precise global measurement of the state of the electorate opinions and so constitute an ideal playground for application of the statistical physics tools to model the opinion dynamics (see Refs. [1–14] and references therein). Analysis of electorate dynamics is one of the key problems for forecasting the results of voting [11]. In this way, it was found that the distributions of the number of candidates according to the number of votes they received in Brazilian, Indian, and Mexican elections can be reproduced with simple spin dynamic models [9,16–18]. The role of inflexible minorities in the breaking of democratic

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doi:10.1016/j.physa.2011.05.034

opinion dynamics was studied in Ref. [10]. An interesting result was obtained in a three-candidate voter model: an initial damage and suppression of one candidate may later lead to an enhancement of the same candidate [19].[1]

In general, a spin dynamic model consists of N agents that are fixed on network vertices. Each agent can either assume the state with spin $s = +1$ or $s = -1$, or behave in the neutral state with $s = 0$ [15]. In the context of social dynamics, spins of different signs can be associated with active agents of two different parties, while uncommitted voters are associated with $s_i = 0$.[2] The system evolves according to a specific dynamic rule and generally tends to a stable magnetic state [15].

The social dynamic rules can be divided in two groups [23]. Those in which individuals form their beliefs based on the opinions of their neighbors in a social network of personal acquaintances (see, for example, Refs. [3,6,7,11]), and those in which, conversely, network connections form between individuals of similar beliefs [24,25].[3] Various rules of neighbors' influence have been proposed to model the opinion formation (spin dynamics), among which are the voter model (VM) [26–28], the Sznajd model [6], the bounded confidence models [7,21,29,30], the social impact models [5], the majority rule (MR) models [3], and the "vacillating" VM [31,32], among others (see, for review Refs. [3,4]). Each of these models belongs to a well defined universality class determined by the system dimensionality (d), the conservation laws, the symmetry of the order parameter, the range of the interactions, and the coupling of the order parameter to conserved quantities [33]. Each class of universality is characterized by a set of critical exponents governing the behavior of the spin model [33–35].

In the paradigmatic VM [27], at each time step, a randomly selected node adopts the opinion of a randomly-chosen neighbor.[4] This step is repeated until the system reaches consensus. The VM node-update dynamics conserves the ensemble average magnetization in a regular lattice. So, the probability that the system eventually ends with all positive (negative) spins equals the initial density of positive (negative) spins in all spatial dimensions. The time to consensus t_s depends on the system size N. In the VM on a two-dimensional lattice the time to consensus scales as $t_s \propto N \ln N$, while $t_s \propto N^2$ in $d = 1$ and $t_s \propto N$ for $d > 2$ [4]. It has been shown that the two-dimensional VM represents a broad class of models for which phase ordering takes place without surface tension [28]. The effect of long-range interactions, such as the shortcuts present in small-world networks on the ordering process of the VM was studied in Ref. [36]. The introduction of noise drastically affects the VM dynamics [37]. In particular, VM with noise does not converge to a state of complete order in the thermodynamic limit [37]. The authors of Ref. [38] have introduced an additional rule in the VM termed as the latency: after a voter changes its opinion, it enters a waiting period of stochastic length where no further changes take place. In this case the average magnetization is not conserved, and the system is driven toward zero magnetization, independently of initial conditions [38]. The authors of Ref. [39] have proposed a variant of the voter model by introducing the social diversity in the evolution process. The VM update rule has also been used to study the dynamics of systems with more than two opinion states [19,40,41]. The cyclically dominated three-state voter model on a square has been extended by taking into consideration the variation of Potts energy during the nearest neighbor invasions [41], for study of the effect of surface tension on the self-organizing patterns maintained by the cyclic invasions. It was found that the model undergoes a continuous phase transition as this parameter is varied, from a regime in which opinions are arbitrarily diverse to one in which most individuals hold the same opinion [41]. Castellano et al. [42] have suggested a nonlinear variant of the VM, the q-voter model, in which q neighbors (with possible repetition) are consulted for a voter to change opinion. If the q neighbors agree, the voter takes their opinion; if they do not have a unanimous opinion, still a voter can flip its state with probability δ.[5] A simple modification of VM was suggested by Sznajd–Weron and Sznajd [6]. It is based on the principle that if two friends share the same opinion, they may succeed in convincing their acquaintances of their opinion ("united we stand, divided we fall").[6] Fortunato [21] has proposed the extension of the Sznajd model with continuous opinions, based on the criterion of bounded confidence.[7] Masuda et al. [43] have introduced the heterogeneous (HVN) and partisan (PVM) voter models. In the HVM each agent has its own intrinsic rate to change state, reflective of the heterogeneity of real people, whereas in the PVM each agent has an innate and fixed preference for one of two possible opinion states [43].[8] Some other modifications of the classic VM rule have been also suggested (see Ref. [4] and references therein) to model non-equilibrium systems.

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1 While surprising at first glance, this phenomenon was observed during the electoral campaign in the 2006 Mexican presidential election [20].

2 In models with continuous opinion, each individual has, at least initially, its own attitude/opinion, such that s_i are real numbers and a "confidence bound" ε is introduced such that two agents are compatible if their opinions differ from each other by less than ε, i.e. $|s_i - s_{i+1}| < \varepsilon$ [7,21]. Furthermore, the opinions can be represented by vectors with real-valued components [22].

3 Holme and Newman [23] have suggested the combination of these two processes within a simple model with a single parameter controlling the balance between two procedures.

4 Voter dynamics was first considered by Clifford and Sudbury [26] as a model for the competition of species and named the "voter model" by Holley and Liggett [27] for the very natural interpretation of its rules in terms of opinion dynamics.

5 This model illustrates how apparently innocuous changes in the microscopic dynamics can lead to different types of collective phenomena, and in particular to different paths to reach consensus [42].

6 In the original Sznajd's algorithm, if a randomly chosen pair of neighboring agents are in the same state $s_i = s_{i+1}$, their neighbors $i-1$ and $i+2$ adopt the same state, whereas if $s_i = -s_{i+1}$, each agent of the pair "imposes" its opinion to the neighbor of the other agent of the pair, so $s_{i-1} = s_{i+1} = -s_i = -s_{i+2}$ [6]. With these rules there are two possibilities of a final state: Complete consensus (perfect ferromagnetic) or a perfect splitting of the community in two factions, with exactly half of the agents sharing either opinion (perfect antiferromagnetic). However, in most successive studies on the Sznajd model the second rule has usually been neglected, such that a consensus becomes the only possible final.

7 It was found that in the model with continuous opinion and the original Sznajd update rules the bi-polarization is very likely to occur at low values of confidence bound $\varepsilon < \varepsilon_C \cong 0.5$ [21].

8 Both, the partisan and heterogeneous VM rules move a finite size system toward a consensus, but much slower than the classic VM [43].