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Endogenous fluctuations in an endogenous growth model: An analysis of inflation targeting as a policy☆

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ABSTRACT

We implement two different monetary policies – an inflation targeting policy as well as a cash reserve requirement – in a monetary endogenous growth overlapping generations model characterized by production lags and analyse the growth dynamics that emerge from this framework. The growth process is endogenized by allowing productive government expenditure on infrastructure, complementing the lagged private capital input. In the presence of these monetary policies, we show that multiple equilibria emerge along different growth paths, with the low-growth (high-growth) equilibrium being unstable (stable) and locally determinate (locally indeterminate). In addition, we highlight the possibility of convergent or divergent endogenous fluctuations and even topological chaos around the high-growth equilibrium in the growth path where the monetary authority follows a high inflation targeting regime. Conversely, when the monetary authority follows a low inflation targeting regime, oscillations do not occur around either the low-growth or high-growth equilibrium. Moreover, a strictly non-negative cash reserve requirement is a necessary and sufficient condition to initiate the growth process.

1. Introduction

"Economic progress, in capitalist society, means turmoil." – Joseph A. Schumpeter (1942)[1]

In this paper, we develop a monetary endogenous growth overlapping generations model with inflation targeting, characterized by production lags, to analyse growth dynamics. The growth process is endogenized by allowing for productive government expenditure on infrastructure[2] in the vein of Barro (1990). Money is introduced through an obligatory reserve requirement, set by government and imposed on the banking system, which otherwise operate in a perfectly competitive environment.[3] Moreover, instead of the standard money growth rule set by the monetary authority, we assume that the monetary authority implements an inflation-targeting (IT) regime across both generations. This assumption introduces growth dynamics into the model that yield results much richer than those presented by related works of Chetty and Ratha (1996), Michel (1993), and Gupta (2011), which we discuss below in detail.

The motivation for this paper stems largely from the initial findings in Michel (1993), the subsequent analysis detailed in Chetty and Ratha (1996) and finally, Gupta (2011) who studies growth dynamics in a similar OLG model with no money or productive government expenditure, but rather using firm-specific and per capita

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E-mail address: rangan.gupta@up.ac.za (R. Gupta).
[1] Capitalism, Socialism and Democracy.
[2] The infrastructure referred to here, comprises typical government activities that stimulate firm activities, such as highways, railways, water systems, power systems, police and fire services, and courts. For simplicity, we will assume that there is no congestion as in Barro and Sala-i-Martin (1992) and the infrastructure provided is a non-rival public good for firms.
[3] This is a standard treatment of money in the literature. See Bittencourt, Gupta, and Stander (2014) as well as Gupta and Stander (2017) and the references cited therein, for a detailed account of the related literature and the motivation for this treatment of money. Alternatively, we could have introduced money through a cash-in-advance constraint as in Kudoh (2007). However, our results would remain unchanged, as long as we either assume the parameterization of the utility function is such that either savings is positively related to the interest rate or is inelastic with respect to the interest rate. However, in the former case, where the savings function is positively related to the interest rate, no closed-form solutions are obtained and we need to linearize equations. Introducing money through the cash-reserve requirement allows us to avoid these complications without any loss of generality.

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economy-wide capital input. In this setting, endogenous convergent fluctuations emerge due to the lagged production input, where the speed of the convergence is determined by the marginal product of labour given the initial value of the gross growth rate. The Chetty and Ratha (1996) findings, that growth is feasible in an OLG model with production lags if the productivity of capital is sufficiently high and borrowing is for capital services only, and not to finance wages in advance as well, is not only contradictory to the well-known Jones and Manuelli (1990) result – that growth rates in an OLG model with convex production is bounded above by zero, and hence, sustained growth is not feasible – but it also highlight the crucial impact of the time-structure of production on the economic growth rate.

Due to the technology used in Chetty and Ratha (1996) being of the Solow (1956)-type, and hence, the results pertaining only to exogenous growth processes determined by the population growth, there was no implication for endogenous growth processes in a setting where the time-structure of the production process is altered. In this sense, the Chetty and Ratha (1996) findings suffer from the same drawback as any Solow (1956)-type model, in that it lacks the ability to explain the non-zero growth in the per-capita standard of living in steady-state observed in the data. The Gupta (2011) findings do extend the impact of lagged production inputs to endogenous growth processes, however the economic environment presented therein included a Romer (1986)-type production technology and did not include a role for money, and hence, a role for monetary policy.

Complimentary to the theoretical motivation, Day (1983) provides a more philosophical reminder:

> ...concerned with the emergence of erratic fluctuations in economic growth processes, fluctuations of a highly irregular or unstable nature termed "chaotic" in the mathematical literature, that emerge *endogenously* [own emphasis] through the interplay of technology, preferences, and behavioural rules alone, with *no exogenous interference* [own emphasis] from stochastic shocks.

The model developed by us, albeit in an OLG framework, could be compared to the "time-to-build" model of Kydland and Prescott (1982), and the "vintage capital" infinitely-lived, representative agent model of Benhabib and Rustichini (1991). The first comprehensive survey on indeterminacy of equilibriums in OLG models, was provided by Woodford (1984). Quite clearly, the endeavour to understand the impact of production lags and growth dynamics across two different strands of the literature is not new.[4] Though, the use of lagged inputs is not as prevalent in analysing growth processes as perhaps it should be, especially in the theoretical[5] growth literature where the use of contemporaneous capital and labour inputs are almost the standard treatment. This could perhaps largely be attributed to a concern first shared in Fabbri and Gozzi (2008), that "...the introduction of vintage capital allows to explain some growth facts, but strongly increases the mathematical difficulties". Fabbri and Gozzi (2008) then proceed to analyse fluctuations using the Maximum Principle as well as Dynamic Programming.

In contrast, we follow a simpler tract in analysing growth fluctuations that are generated endogenously. The theoretical OLG model presented here is developed along the lines of Gupta (2011). However, we extend this analysis by allowing for the role of money and hence, monetary policy as well as productive government expenditure and the include a role for the banking sector. Furthermore, introducing an inflation targeting regime ties the optimal growth rate to the inflation target, which allows for a deeper understanding of the role of monetary policy on the equilibrium growth path and endogenous fluctuations in the characterized economy.

More closely related to the discussion in this paper are the studies of El-Hodiri, Loehman, and Whinston (1972) who are the first to introduce lags into a model of optimal growth; Benhabib and Day (1982) who characterize wide classes of utility functions which generate erratic dynamics in an overlapping generations OLG model; Reichlin (1986) who uses the Hopf bifurcation theorem to detect stable or unstable equilibrium trajectories where the determining parameter of the bifurcation is the technological externality to the production process; Galor and Ryder (1991) who study the dynamic efficiency of equilibria in an OLG model; Michel (1993) who shows that the growth rate of the economy oscillates on a transitional path if capital used in production is *not* contemporaneous; Matsuyama (1999) who finds that economic fluctuations are driven by periods of high investment or periods of high innovation; Kitagawa and Shibata (2001) who shows that investment gestation lags causes permanent cyclical movements in the *level* of national income without a production technology; and lastly Kitagawa and Shibata (2005) who develop a simple OLG model with investment gestation lags and find that if the production technology is of the AK-type, the existence of investment lags causes permanent cyclical fluctuations in the growth rate. But, as indicated above, we deviate primarily in highlighting the importance of monetary policy, and in particular, the role played by the size of the inflation target in causing growth fluctuations.

The rest of this paper is organized as follows: Section 2 defines the economic environment that comprises of representative consumers, banks, producers and a productive government. In these sections, we also solve the optimization problem of each of the agents. Once the optimization is solved, in Section 3, we characterize the equilibrium of the economy along a balanced growth path (BGP), along which all real variables grows at the same rate. In Section 4, we then examine the growth dynamics of the characterized economy, highlighting the possibility of multiple equilibria, indeterminancy and growth fluctuations to the extent that chaos might emerge under certain conditions. Section 5 presents associated policy implications of our results, whereby we also indicate the ways to avoid chaotic growth dynamics by the appropriate choice of the size of the inflation target. Finally in Section 6, we offer some concluding remarks.

2. The economic setting

Time is divided into discrete segments and indexed by $t = 1, 2, \ldots$. The principal economic activities are: (i) every possible two-period lived overlapping generation consumer/labourer, receives a positive young-age labour endowment of unit one, but retires and consumes only when old.[6] Thus, at time point t, there are two coex-

[4] Since the seminal work of Kalecki (1935) on production lags and business cycles, the study of the impact of different types of lags (including production lags) has been the intense focus of a vast literature. Note that, although Jevons (1835–1882) has been credited with opening the discussion on production lags and cycles in output whilst it was more likely Hearn (1826–1888) who first broached the concept, it was in fact Kalecki who formally analysed this first in his 1935 paper, "A Macrodynamic Theory of Business Cycles" published in *Econometrica*. Noteworthy contributions analysing the impact of various forms of lags on theoretical issues relating to prices, markets, investments, and cycles, among others include those of Day (1983), Goodwin (1947), Grandmont (1985), Kydland and Prescott (1982), May (1976), Mayer (1960) and again, Goodwin (1990).

[5] In the empirical literature, however, the standard inclusion of lags of the growth process in the modelling of growth dynamics could be seen as a *de facto* attempt to account for the impact of lagged inputs on the growth process.

[6] This assumption ensures tractability and makes the analysis independent of the consumers utility function, as it abstracts from the consumption-savings decision. See Woodford (1984) for initial discussion on this, although this assumption is frequently used in the OLG literature. See among others, Cazzavillan (1996) and

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