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GMM estimation of the spatial autoregressive model in a system of interrelated networks

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ABSTRACT

This paper considers efficient estimation of spatial autoregressive models in a system of interrelated networks. An example describes a market situation with several chain stores competing against each other. The strategy of a store in the chain does not only involve coordination with the other stores in the same chain, but also competition against opponent stores in other chains. To estimate the system, we extend the generalized method of moments framework based on linear and quadratic moment conditions proposed by Lee (2007) and Lin and Lee (2010). We show that under some regularity assumptions the proposed GMM estimator is consistent and asymptotically normal. We derive the best GMM estimator under normality and propose a robust GMM estimator against unknown heteroskedasticity. Monte Carlo experiments are conducted to study the finite sample performance of the GMM estimation. We also provide an empirical application of the model on the spatial competition between chain stores in the market of prescription drugs.

1. Introduction

Spatial econometric models have attracted considerable interest in various fields of economics, including urban, labor, international and development economics, among others. One of the most widely used spatial models is the single equation model introduced by Cliff and Ord (1973, 1981), which is referred to a spatial autoregressive (SAR) model, see, e.g., Anselin (1988). In this paper, we consider an extension of the SAR model to the study of interactions across networks. In particular, we consider the estimation of network outcomes in a system of interrelated networks. The model describes a market situation with several chain stores competing against one another. Each chain's outcome, e.g., prices, is represented by a vector. In this market structure, stores' pricing strategy could be more complicated than that in a simple oligopolistic market. It includes not only the traditional price competition among chains, but also relationships among prices of stores in the same chain. In Appendix E, we use a simple price competition model to study such a market structure where stores are from two different chains. We show that in the equilibrium, the strategy of a store in a chain does not only involve competition against opponent stores of other chains, but also coordination with other stores in the same chain. Therefore, we can set up a SAR equation for each chain, but those equations need to be estimated simultaneously because each equation contains spatial lag terms (reactions) to other chains in the market. Aside from spatial correlations among outcomes of chains, the error terms are also assumed to be spatially correlated among chains, which contributes another source of spatial dependence.[1]

In this model, as the spatial lag terms may have different coefficients, it will be computationally intensive to implement the method of maximum likelihood (ML) even when the disturbances are normally distributed due to the complicated Jacobian transformation. This is so, especially for models with large sample sizes. Moreover, Ord's (1975) device for a single equation SAR model cannot be extended to this model due to multiple spatial lags on outcomes. Similar situation occurs in the high order SAR model, see, e.g., the discussion and references in Lee and Liu (2010). Furthermore, it is computationally difficult to impose general stability conditions via unknown parameters of the system. An issue similar to that for the high order spatial lags model as pointed out in Elhorst et al. (2012). Therefore, in this paper, we introduce a generalized method of moments (GMM) estimation of the model, based on modified linear and quadratic moments in Lee (2007). Specifically, we demonstrate that our GMM estimators are consistent

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and asymptotically normal, and we also derive the most efficient GMM estimator under the normality assumption and show that it is asymptotically as efficient as the ML estimator. In order to deal with possible spatial correlations in error terms, we generalize some of the specification in Lee and Liu (2010) to fit the multiple network setting. In addition, while ML approach might be inconsistent in the presence of unknown heteroskedastic errors, GMM methods can be easily modified for consistent estimation under such a scenario.

There has been some attempt on the specification and estimation of multiple equation systems with spatial dependence in the literature. Kelejian and Prucha (2004) introduce a very general simultaneous equation SAR (SESAR) model, incorporating spatial lags in dependent variables and allowing for spatial correlation in disturbance terms. They consider two stage least square (2SLS) and three stage least square (3SLS) estimation methods for such a model. Baltagi and Deng (2015) extend the model to fit panel data by deriving a 3SLS estimator for SESAR model with random effects. Baltagi and Bresson (2011) employ the ML method to estimate a spatial seemingly unrelated regression panel model. The disturbances allow spatial error components. They also propose joint and conditional Lagrange multiplier tests for the presence of spatial correlation and random effects. Liu (2014) considers identification and estimation of social network models in a system of simultaneous equations. 2SLS and 3SLS estimations with many instrumental variables (IV) and bias correction procedures are provided. Liu and Saraiva (2017) proposes a GMM estimation framework for the SAR model in a system of simultaneous equations with heteroskedastic disturbances. Besides linear moment conditions, their GMM method also utilizes quadratic moment conditions based on the covariance structure of model disturbances within and across equations. Yang and Lee (2017) study the identification and quasi maximum likelihood (QML) estimation of a simultaneous equations spatial autoregressive model which incorporates simultaneity effects, own-variable spatial lags and cross-variable spatial lags as explanatory variables, and allows for correlation between disturbances across equations. Although the models considered in these existing works are different in some ways, they all consider multivariate outcomes of each spatial unit. Thus, each of the simultaneous equations presents a structure for an outcome variable and, therefore, each equation has the same sample size. In a typical SESAR model, there might be a single network linking the spatial units. Our model differs from theirs in that each of the multiple equations may have different sample sizes (multiple chains may have different number of stores, which is often the case). In our model, each network has its distinctive feature for an equation, and multiple networks generate multiple equations. Each network may have a different size in general. As for the estimation method, our GMM estimator utilizes both linear moment conditions based on the orthogonality condition between the IV and model disturbances, and quadratic moment conditions based on the covariance structure of model disturbances (similar to those proposed by Lee, 2007, Lee and Liu, 2010, Lin and Lee, 2010, and Liu and Saraiva, 2017), thus is more efficient relative to the 2SLS and 3SLS.

The paper is organized as follows. In Section 2, we introduce the SAR model within a system of interrelated networks and study the parameter space and identification conditions under the traditional likelihood framework. Computational issues on the ML estimation are discussed. We establish identification of the model and propose a computationally tractable GMM estimation approach in Section 3. Section 4 investigates consistency and asymptotic distribution of GMM estimators. Section 5 derives the best selection of moment functions under normality and discusses efficiency properties of the best GMM (BGMM) estimator. A robust GMM estimator against unknown heteroskedasticity is proposed in Section 6. Section 7 provides Monte Carlo results on finite sample properties of the GMM estimators. In Section 8, we provide an empirical application of the model on spatial competition between chain stores in the market of prescription drugs. Section 9 concludes. All the proofs of the results are collected in the Appendices.

2. The model, parameter space and identification

The model under consideration with r interrelated networks is specified in its general form as

$$
\begin{aligned}
Y_{1,n_1} &= \lambda_{11} W_{11} Y_{1,n_1} + \lambda_{12} W_{12} Y_{2,n_2} + \cdots + \lambda_{1r} W_{1r} Y_{r,n_r} + X_{1,n_1}\beta_1 + U_{1,n_1} \\
Y_{2,n_2} &= \lambda_{21} W_{21} Y_{1,n_1} + \lambda_{22} W_{22} Y_{2,n_2} + \cdots + \lambda_{2r} W_{2r} Y_{r,n_r} + X_{2,n_2}\beta_2 + U_{2,n_2} \\
&\vdots \\
Y_{r,n_r} &= \lambda_{r1} W_{r1} Y_{1,n_1} + \lambda_{r2} W_{r2} Y_{2,n_2} + \cdots + \lambda_{rr} W_{rr} Y_{r,n_r} + X_{r,n_r}\beta_r + U_{r,n_r}
\end{aligned}
\tag{1}
$$

with

$$
\begin{aligned}
U_{1,n_1} &= \rho_{11} M_{11} U_{1,n_1} + \rho_{12} M_{12} U_{2,n_2} + \cdots + \rho_{1r} M_{1r} U_{r,n_r} + \varepsilon_{1,n_1} \\
U_{2,n_2} &= \rho_{21} M_{21} U_{1,n_1} + \rho_{22} M_{22} U_{2,n_2} + \cdots + \rho_{2r} M_{2r} U_{r,n_r} + \varepsilon_{2,n_2} \\
&\vdots \\
U_{r,n_r} &= \rho_{r1} M_{r1} U_{1,n_1} + \rho_{r2} M_{r2} U_{2,n_2} + \cdots + \rho_{rr} M_{rr} U_{r,n_r} + \varepsilon_{r,n_r}
\end{aligned}
$$

where for $i, j = 1, \ldots, r$, Y_{i,n_i} is a $n_i \times 1$ vector of outcomes of all the n_i individuals from group i; $X_{i,ni}$ is an $n_i \times k_{x_i}$ matrix of nonstochastic regressors for group i; W_{ii}'s and M_{ii}'s are $n_i \times n_i$ nonstochastic network links matrices for within group individuals, measured in terms of physical or economic distances between individuals in group i, and W_{ij}'s and M_{ij}'s with $i \neq j$ are $n_i \times n_j$ nonstochastic network matrices for between group individuals, which capture the spatial correlations between individuals from different groups i and j; U_{i,n_i}'s are the disturbances vectors with spatial correlations; elements of ε_{i,n_i} are zero mean *i.i.d.* disturbance such that $\varepsilon_{i,n_i} \sim (0, \sigma_i^2 I_{n_i})$, and ε_{i,n_i} and ε_{j,n_j} for $i \neq j$ are also independent.

Equation (1) models a system of outcomes with r interrelated networks. In industrial organization for an example, it may describe a market with r competitors where each competitor i has n_i chain stores. In the case $r = 2$, it fits conveniently in the case of a duopoly. For instance, Y_{1,n_1} may represent a vector of prices or quantities of Wal-Mart stores and Y_{2,n_2} represents a vector of prices or quantities of K-Mart stores. W_{11}, W_{22} are, respectively, spatial weight matrices for Wal-Mart and K-Mart stores designated in terms of distances between stores. Similarly, W_{12} and W_{21} denote the distances between rival stores. Unlike W_{11} and W_{22}, W_{12} and W_{21} are respectively $n_1 \times n_2$ and $n_2 \times n_1$ matrices rather than square matrices when $n_1 \neq n_2$.[2]

We can write the model in the following form:

$$
\begin{pmatrix} Y_{1,n_1} \\ Y_{2,n_2} \\ \vdots \\ Y_{R,n_R} \end{pmatrix}
=
\begin{pmatrix}
\lambda_{11} W_{11} & \lambda_{12} W_{12} & \cdots & \lambda_{1r} W_{1r} \\
\lambda_{21} W_{21} & \lambda_{22} W_{22} & \cdots & \lambda_{2r} W_{2r} \\
\vdots & \vdots & \ddots & \vdots \\
\lambda_{r1} W_{r1} & \lambda_{r2} W_{r2} & \cdots & \lambda_{rr} W_{rr}
\end{pmatrix}
\begin{pmatrix} Y_{1,n_1} \\ Y_{2,n_2} \\ \vdots \\ Y_{R,n_R} \end{pmatrix}
+
\begin{pmatrix}
X_{1,n_1} & 0 & \cdots & 0 \\
0 & X_{2,n_2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & X_{r,n_r}
\end{pmatrix}
\begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_r \end{pmatrix}
+
\begin{pmatrix} U_{1,n_1} \\ U_{2,n_2} \\ \vdots \\ U_{r,n_2} \end{pmatrix},
$$

or equivalently,

[1] Our equations may also contain spatial lags of the exogenous variables. For simplicity, if they are included, they are treated as subset of exogenous variables. Such a treatment does not have specific theoretical econometric irregularity for a SAR model.

[2] Due to distance in this example, we have $W_{12} = W_{21}'$. However, this condition can be relaxed in a more general setting with asymmetric W_{ij}'s.