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Optimal quality factor for tuning forks in a fluid medium

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ABSTRACT

Resonator based sensing devices operated within a fluid medium often require maximal quality factors in order to enhance their performances. We present a tuning fork shape optimization formalism based on both analytical and numerical approaches, and identify the geometry with the lowest possible losses. We also report on the fabrication of a homemade tuning fork based on this optimization process, and experimental measurements show a quality factor of Q = 41000 in air at atmospheric pressure. This value represents, to the best of our knowledge, the record quality factor for a flexural resonator in ambient air.

1. Introduction

Flexural mechanical resonators have been extensively used in the past century, and are most often operated in vacuum in order to minimize any coupling with the surroundings that could affect their excellent resonance characteristics. Some applications however require their complete immersion within a fluid medium. It is for example used for highly sensitive chemical sensing platforms [1], but also for in-situ force measurements such as the photo acoustic detection [2], the resonant optothermoacoustic detection [3] or precise temperature measurements [4].

For resonators operated in a fluid, the surrounding fluid strongly modifies the characteristics of the resonance and can dramatically reduce the sensors performances compared to vacuum. Those performances are however usually closely related to the amount of damping experienced by the resonator. It is now established that the damping occurring within the fluid can be attributed to several mechanisms, among which viscous damping and acoustic radiation damping are the most significant. These two effects, originating both from a fluid-structure interaction but relying on two distinct physical properties of the fluid that are viscosity and compressibility, behave differently with respect to the various dimensions of the resonator geometry. A shape optimization could therefore enhance fluid sensor performances.

Analytical modeling should be preferred over numerical treatments since the dimensions of the resonator can vary over possibly several orders of magnitude. Shape optimization is all the more complex that a wide number of resonator's shapes have appeared over the years, each one requiring specific models. In the following, we only consider the case of tuning forks since they are one of the most popular geometry for sensing applications. A similar study has been proposed for single beam resonators, but the approach remains theoretical and only applies to infinitely thin cantilevers [5,6].

Although our method applies to any tuning fork oscillating within any viscous fluid, we present a quality factor optimization in air at atmospheric pressure for a clamped-free quartz tuning fork class. We also fabricate a tuning fork close to the optimal, and its experimental quality factor is compared with the theoretical prediction.

2. Optimal tuning fork

2.1. Analytical expression for the quality factor

We consider a tuning fork vibrating in an infinite and compressible fluid medium. Let ρb denote the tuning fork volumetric mass density, e the dimension of the prongs along the displacement direction, l the dimension along the displacement orthogonal direction, g the gap between the two prongs and ω the angular frequency of the vibration. Concerning the fluid, μ denotes be the dynamic viscosity, ρf the fluid volumetric mass density and c the speed of sound. A schematic of the situation can be found in Fig.1.

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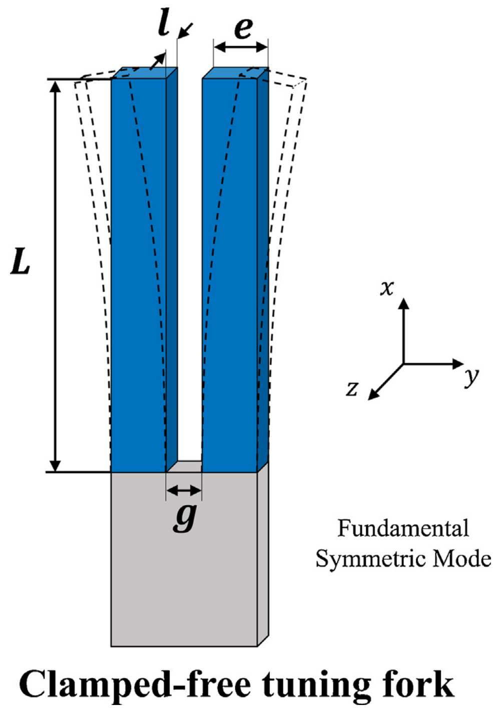

Fig. 1. Schematic of a clamped-free tuning fork operated on its first in-plane symmetric mode of vibration.

The total quality factor of the tuning fork can be written as the sum of its different contributing effects, according to the usual additive rule valid for independent damping sources:

$$\frac{1}{Q_{tot}} = \frac{1}{Q_s} + \frac{1}{Q_t} + \frac{1}{Q_v} + \frac{1}{Q_a}. \quad (1)$$

The quality factor Q_S represents the anchor losses. The actual value of this factor in a resonator greatly depends on the efforts spent in the choice of the anchors locations with respect to the vibrating mode. It is known that very high support quality factors $Q_S>10^5$ can be achieved if the tuning fork is operated on its symmetric in-plane mode of operation, especially if g is small compared to the beam's length. For that reason, we restrain our optimization to this mode of vibration, and consider that Q_S is infinite.

The quality factor Q_t is the thermoelastic damping. We include a known expression in our optimization [7]

$$Q_t = \frac{C_p \rho_b}{E\alpha^2 T_0}\left(\frac{6}{\xi^2} - \frac{6}{\xi^3}\frac{\sinh \xi + \sin \xi}{\cosh \xi + \cos \xi}\right)^{-1}, \quad (2)$$

where E is the material's Young modulus, α its thermal expansion coefficient, T_0 its temperature, C_p its heat capacity, κ its heat conductivity and $\xi = e\sqrt{\omega \rho_b C_p / 2\kappa}$.

The quality factor Q_v is the quality factor of the beam associated to the viscosity of the surrounding fluid. Theoretical expressions have been reported for any rectangular cross section shape [8,9], and a simplified expression can be given for high beta parameters $\beta = \rho_f \omega l^2 / 4\mu \gg 1$ as

$$Q_v = \left[\left[\frac{1}{\rho_b l} + \frac{\pi}{\rho_b e}\right]\sqrt{\frac{2\rho_f \mu}{\omega}} + \frac{8\mu l^2}{\rho_b e \omega g^3}\right]^{-1}. \quad (3)$$

The latter expression contains the sum of two contributions: the first one represents viscous damping in the unbounded fluid case, while the second one is due to a possible squeeze film damping caused by the tuning fork prongs proximity. We choose in the optimization process a somewhat arbitrary value $g = l$, which represents a good trade-off between a gap wide enough to neglect squeeze film damping and minimum anchor losses by the tuning fork. In the following, exact expressions from Ref. [8] are used for the viscous damping quality factor.

The quality factor Q_a is associated to the acoustic losses. We recently reviewed the different expressions valid for tuning forks vibrating on their in-plane mode of vibration [10], and an expression using Schmoranzer model [11] has been shown to be relevant in our case:

$$Q_a = 2\frac{\rho_b}{\rho_f}\frac{e}{l}\frac{1}{\sum_{m=0}^{m=+\infty}\frac{2}{1+\delta_{0,m}}\left[J_{2m}\left(k\left(e+\frac{g}{2}\right)\right) - J_{2m}\left(\frac{kg}{2}\right)\right]^2}. \quad (4)$$

In the latter expression, J_m is the first order Bessel function, δ designates the Kronecker symbol and $k = \omega/c$ is the vibration wavenumber.

2.2. Optimal tuning fork as a function of its length

The space of parameters is constituted by the three beam dimensions (e,l,L) and constrained as follows:

- The prong cross section dimensions (e,l) have to be smaller that at least $L/2$. This assumption is required in order to use the acoustic quality factor expression. Moreover, the Euler-Bernoulli bending theory approximations become irrelevant beyond that limit, and the quality factor expressions for viscous and thermo elastic damping can show severe discrepancies.
- The prongs length has to be in a region of interest, and we choose $3\mu m < L < 0.1m$.

For a given length, we can plot the values of the quality factor as a function of the cross section dimensions. The quartz young modulus used is E=78.3GPa and its density is $\rho_b = 2650 kg.m^{-3}$. We obtain typical maps displayed in Fig. 2, from which we can extract the optimum cross section. We remind that the first resonance frequency is given by $\omega_0 = \left(\alpha_0/L\right)^2 e\sqrt{E/(12\rho_b)}$ with $\alpha_0 \sim 1.875$according to Euler-Bernoulli theory of bending; therefore the beam resonance frequency increases linearly with the prongs width.