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Sufficient conditions for determinacy in a class of Markov-switching rational expectations models ☆,☆☆

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ABSTRACT

Markov-switching rational expectations (MSRE) models can bring out fresh insights beyond what linear rational expectations models have done for macroeconomics, as noted and predicted by Davig and Leeper (2007) and Farmer et al. (2009), among others. However, a lack of tractable methodological foundations may have hindered researchers from uncovering the salient features of MSRE models. This study proposes a solution method and derives very tractable sufficient conditions for determinacy and indeterminacy in the mean-square stability sense in general MSRE models with lagged endogenous variables. These tasks are accomplished by extending the forward method of Cho and Moreno (2011) developed for linear rational expectations models to MSRE models. We apply our methodology to a New-Keynesian model subject to regime-switching in monetary policy and find some unforeseen but intuitive determinacy results.

1. Introduction

Markov-switching, a popular modeling technique in time-series econometrics, has only recently been applied to structural rational expectations (RE) macroeconomic models. This new class of models, referred to as Markov-switching rational expectations (MSRE) models, can bring out fresh insights about the time-varying behaviors of policy authorities and private agents. For instance, Davig and Leeper (2007) provide a new perspective — expressed as the long-run Taylor principle (LRTP) — in which the possibility of regime-switching between passive and active monetary policy stances in an otherwise canonical New-Keynesian model can expand the parameter space over which a unique bounded equilibrium exists. Indeed, Liu et al. (2011) and Bianchi (2013) find a recurrent pattern of US monetary policy switching over an active and a passive regime using an estimated New-Keynesian model that allows shifts in the monetary policy reaction coefficients and shock volatilities. Baele et al. (2015) also show that the US economy has experienced multiple shifts in both monetary policy

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and shock volatilities, and provide new estimates of the beginning and end of the Great Moderation period. Using a similar model, Davig and Doh (2014) explain the low–high–low pattern of inflation persistence observed in the US economy. Bianchi and Melosi (2013) develop a theoretical model from the perspective of the fiscal theory of price level in which agents learn about shifts in fiscal policy over time to account for the link between inflation and fiscal imbalances. They find that dormant shocks explain the high and volatile US inflation observed in the 1970s.

The time-varying behavior of private agents and the formation of their expectations can also be studied in the MSRE framework. Gordon and St-Amour (2000) show that the regime-switching preferences of households can account for observed US financial market data such as sharp swings in asset prices. Liu et al. (2009) develop a New-Keynesian model that allows regime-switching in both monetary policy and the optimal price-setting behavior of firms, and emphasize the importance of modeling the connection of expectation formation mechanisms to policy regime. The optimal monetary policy of a central bank facing such regime-shifting behaviors of private agents is also of great importance, as emphasized by Davig (2007). Thus, many important macroeconomic issues can be analyzed both theoretically and empirically in the MSRE framework.

While some progress has been made regarding the class of RE solutions in recent years, the following question, which is essential for understanding MSRE models and their equilibrium properties, has not been answered well in the literature. Under what circumstances would a MSRE model have a determinate — unique stable — solution? Farmer et al. (2009) provide necessary and sufficient conditions for determinacy in a class of purely forward-looking MSRE models in the mean-square stability (MSS) sense by identifying all of the sunspot solutions and requiring all of them to be unstable. These conditions are, however, difficult to examine in practice because the set of sunspot solutions has a continuum of members sharing the same functional form, and its dimension increases exponentially with the size of the model and the number of regimes. Farmer et al. (2011) develop a numerical algorithm for solving fundamental solutions to MSRE models with predetermined variables.[1] Thus, it can identify indeterminacy if more than one solution is found to be stable. However, it is not known how many fundamental solutions exist for a MSRE model because of its inherent non-linearity, and therefore determinacy cannot be identified. By contrast, Davig and Leeper (2007) propose conditions for the uniqueness of a bounded solution to a model similar to that of Farmer et al. (2009). However, their conditions apply to what is known as a quasi-linear system of the underlying MSRE model. In any case, no tractable determinacy conditions for MSRE models have been developed under any concept of stability so far.

This study establishes sufficient conditions for determinacy in the MSS sense for a class of general MSRE models. Our results extend those of Farmer et al. (2009) by expanding the class of models to include lagged endogenous variables and by deriving sufficient conditions that are easy to verify. Moreover, the resulting determinate solution is straightforward to compute. We also derive sufficient conditions for indeterminacy that rely on a numerical procedure.

The key idea of our methodology for identifying determinacy is to connect the property of the forward solution known as the no-bubble condition (NBC) observed by Cho and Moreno (2011) for linear RE (LRE) models and an important property of MSS derived in this paper, and to extend this idea to MSRE models. This approach does not require us to solve for all of the sunspot solutions to establish determinacy, nor does it require us to solve for all of the fundamental solutions, which are both difficult tasks in the MSRE context as mentioned above.

We apply our methodology to a standard New-Keynesian model subject to regime-switching in a monetary policy stance. Our analysis shows that a temporarily passive monetary policy can lead to a determinate equilibrium, as illustrated by Davig and Leeper (2007). However, we also find that if one regime is too active relative to the other, even when both regimes are active, the additional volatility induced by the regime-switching policy can actually lead the economy to indeterminacy.

The remainder of the paper is organized as follows. Section 2 lays out a class of general MSRE models and RE solutions and introduces the concept of determinacy in the MSS sense. Section 3 develops our forward method. Section 4 derives the sufficient conditions for determinacy and indeterminacy. In Section 5, we illustrate how to implement our methodology using a standard New-Keynesian model. Section 6 concludes.[2]

2. Markov-switching rational expectations models

This section presents the class of general MSRE models and the full set of RE solutions and defines determinacy in the MSS sense.

2.1. The class of MSRE models

The class of MSRE models considered in this paper is given by

$$x_t = E_t[A(s_t, s_{t+1})x_{t+1}] + B(s_t)x_{t-1} + C(s_t)z_t, \tag{1}$$

$$z_t = R(s_t)z_{t-1} + \epsilon_t, \quad \epsilon_t \sim (0_{m\times 1}, D), \tag{2}$$

[1] Farmer et al. (2011) use the term MSV solution instead of fundamental solution. These terminologies are used interchangeably in the literature to denote a solution that depends on the minimum state variables. By contrast, McCallum (1983) uses the MSV solution to denote a fundamental solution that satisfies the solution selection criterion he proposed. To avoid confusion, we use the term "fundamental" throughout this paper.

[2] Matlab codes for our methodology and some numerical examples, including the one analyzed in this paper, can be found at http://web.yonsei.ac.kr/sc719.

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