Burdett–Mortensen model of on-the-job search with two sectors ☆

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A B S T R A C T

The focus of this paper is on the steady state of a two-sector economy with undirected search where employed and unemployed workers can search for jobs, both within a sector and between the sectors. As in the one-sector model, on-the-job search generates wage dispersion among homogeneous workers. The analysis of the two-sector model uncovers a property called constant tension that is responsible for analytical tractability. We characterize the steady state in all cases with constant tension. When time discounting vanishes, constant tension yields the endogenous separation rate in each sector as a linear function of the present value for a worker. The one-sector economy automatically satisfies constant tension, in which case the linear separation rate implies that equilibrium offers of the worker value are uniformly distributed. Constant tension also has strong predictions for worker transitions and value/wage dispersion, both within a sector and between the two sectors. When constant tension does not hold, we compute the steady state numerically and illustrate its properties.

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1. Introduction

In a seminal paper, Burdett and Mortensen (1998, BM henceforth) explore on-the-job search as a mechanism for generating job-to-job flows of workers and wage dispersion simultaneously. Their analysis reveals a deep insight about the interaction between competition and search frictions. On-the-job search generates the possibility that a worker may leave a firm for other firms. Facing this possibility, a firm makes the optimal tradeoff between recruiting and retaining a worker, on the one hand, and ex post profit on the other hand. As a result of this tradeoff, equilibrium wage rates must be distributed continuously in an interval, even though all workers and all firms are homogeneous.[1]

The BM model has found wide applications in labor and macroeconomics. It has been used to explain residual wage inequality, i.e., the wage differential among workers that seems difficult to be explained by worker and firm characteristics (van den Berg and Ridder, 1998; Bontemps et al., 2000; Mortensen, 2005). Also, by linking wage dispersion tightly

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[1] In a related model of the product market, Burdett and Judd (1983) generate a non-degenerate distribution of prices by assuming that some buyers can receive two price quotes before selecting one. The outside offer that an employed worker receives in the BM model acts like the second price quote.

to on-the-job search, the BM model offers a natural framework for jointly studying job mobility, wage dynamics, and the sorting pattern between firms and workers (Postel-Vinay and Robin, 2002; Jolivet et al., 2006; Lise and Robin, 2013; Moscarini and Postel-Vinay, 2013). Moreover, the BM model is extended to explain the interaction between job mobility and wage-tenure contracts (Burdett and Coles, 2003).

In this paper, we analyze a BM model with two sectors that compete for the same pool of workers. There are two motivations for this study. One is theoretical and is to understand why the steady state of the BM model is analytically tractable despite that a non-degenerate wage distribution arises endogenously from on-the-job search. To explore the model, we reformulate an offer as the worker value, i.e., the present value delivered to a worker by a wage rate conditional on the worker's optimal separation in the future. This follows recent developments in the search literature (e.g., Burdett and Coles, 2003; Shi, 2009; Menzio and Shi, 2011). With homogeneous workers, dispersion in worker values represents true residual inequality, whereas dispersion in wage rates may not necessarily do so. With two sectors, in particular, a higher wage is not necessarily a better offer to a worker, depending on which sector the offer comes from, but a higher value is always a better offer. Thus, reformulating an offer as the worker value simplifies a worker's transition decision: a worker accepts higher values no matter which sector they come from. With this reformulation, we discover a feature called constant tension that is responsible for analytical tractability and, importantly, this role of constant tension becomes apparent only if one considers a two-sector model (see the description later). The one-sector BM model has constant tension automatically.

Our second motivation is empirical and is to understand how on-the-job search affects equilibrium dispersion and worker mobility between sectors when firms in different sectors compete for workers in the same labor market.[2] As documented by Lee and Wolpin (2006), there are direct transitions of workers between sectors. In a two-sector BM model, these between-sector transitions generate a link between the entire wage distributions in the sectors. This link is consistent with the empirical analysis in Hoffmann and Shi (2011), who show that rising monthly transition rates of employed workers from the non-service sector to the service sector over the last four decades were accompanied by a faster increase in residual wage inequality in the service sector. A two-sector BM model is a natural candidate for organizing these facts. Before enriching the model with worker and firm heterogeneity, it is useful to examine its analytical properties with homogeneous workers and firms first, which we do in this paper.

In our model, as in BM, both employed workers and unemployed workers can search, each firm offers and commits to a constant wage rate over the worker's employment in the firm, and the offer arrival rate from a sector to a worker is exogenous. In contrast to BM, there are two sectors (A and B). A firm can create a vacancy in either sector and a worker can receive an offer from either sector. The two sectors can differ in the value of output produced in a match, in the offer arrival rates, and in the rate of exogenous separation into unemployment. Moreover, the rate at which an employed worker receives an offer from the incumbent sector may differ from the rate of an offer from the other sector. We assume that a job in sector A produces a higher value than a job in sector B, refer to sector A as the high-productivity sector and sector B as the low-productivity sector.

In each sector, there is an interval of offers (of the worker value) that are optimal for a firm. A high offer increases the success in recruiting and reduces the endogenous quit rate of the worker in the future, at the cost of a lower profit flow of the job. All optimal offers maximize expected profit of a vacancy in a sector. In addition, because a firm can locate the vacancy in either sector, expected profit of a vacancy is equalized between the two sectors in the equilibrium. Competition among firms endogenously determines the relative size between the two sectors and the distribution of workers over values within each sector in the steady state. Because workers can receive offers from either sector, the hiring rate of a vacancy depends on the employed distributions of workers over values in both sectors. Similarly, the separation rate of a worker, which includes both exogenous separation and endogenous quits, depends on the offer distributions in both sectors.

A notable property is that the product of the hiring rate of a vacancy and the separation rate of a worker is constant over worker values in the support of the distribution in a sector. We refer to this property as *constant tension* in the sector and show that it makes the steady state tractable. Equivalently, constant tension requires the marginal intensities of hiring and separation to be equal to each other at every worker value in the support of the distribution in a sector. With constant tension, the hiring rate of a vacancy at each offer in a sector is tied to the separation rate at that offer in that sector. As all endogenous elements of expected profit of a vacancy are related exclusively to the separation rate, the condition of equal expected profit of a vacancy solves the separation rate as a function of the offer. Moreover, in the limit where time discounting, r, goes to zero, the solution of the separation rate in each sector is a linear function of the worker value. We examine all (three) cases of the steady state with constant tension.

The first case is the one-sector BM model. An isolated sector automatically has constant tension, because the marginal intensities of hiring and separation must be equal to each other in order to maintain the steady state in a one-sector economy. Constant tension is the source of analytical tractability of the one-sector BM model. Not only can the steady state be solved analytically, but also the equilibrium offer distribution has a strikingly simple form. The linear solution of the separation rate immediately implies that offer values are *uniformly* distributed in the limit $r \to 0$. In contrast, wage rate offers are distributed according to a square-root function, as is well known from BM.

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[2] Beaudry et al. (2012) use a multi-sector search model to show that even without direct employment transitions between sectors or on-the-job search, there is an equilibrium force that affects the industrial composition and inter-sectoral wage differentials. In an extensive empirical analysis they show this force to be quantitatively important.