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A force sensor based on three weakly coupled resonators with ultrahigh sensitivity

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ABSTRACT

A proof-of-concept force sensor based on three degree-of-freedom (DoF) weakly coupled resonators was fabricated using a silicon-on-insulator (SOI) process and electrically tested in 20 μTorr vacuum. Compared to the conventional single resonator force sensor with frequency shift as output, by measuring the amplitude ratio of two of the three resonators, the measured force sensitivity of the 3DoF sensor was 4.9×10^6/N, which was improved by two orders magnitude. A bias stiffness perturbation was applied to avoid mode aliasing effect and improve the linearity of the sensor. The noise floor of the amplitude ratio output of the sensor was theoretically analyzed for the first time, using the transfer function model of the 3DoF weakly coupled resonator system. It was shown based on measurement results that the output noise was mainly due to the thermal–electrical noise of the interface electronics. The output noise spectral density was measured, and agreed well with theoretical estimations. The noise floor of the force sensor output was estimated to be approximately 1.39nN for an assumed 10 Hz bandwidth of the output signal, resulting in a dynamic range of 74.8 dB.

1. Introduction

For the last couple of decades, emerging micro- and nano-scale devices enabled the measurement of forces in the region of pN to μN. Measurement of the forces in this range plays important roles in many different areas, including surface characterization [1], contact potential difference measurement [2], study of biomechanics [3] and cell mechanobiology [4], inertial sensing [5], manipulation of microscale objects [6] and magnetometer for electronic compass [7], among many others.

Among these miniature force sensors, resonant sensing devices are attractive to researchers due to its quasi-digital output signal and high accuracies [8]. The conventional approach employs a single degree-of-freedom (DoF) resonator; when an external force is exerted on the resonator, the stiffness changes while the mass remains the same, leading to a frequency shift [9]. The challenge to improve the performance of the force sensor, aiming to sense smaller forces motivates research in alternative sensing paradigms.

One promising approach, which couples two identical resonators with a spring much weaker than that of the resonators, is to form a 2DoF system [10]. This approach utilizes a *mode localization* effect which was first described in solid-state physics by Anderson [11]. When a small perturbation is applied on one of the resonators, the mode shapes of the system change. It was demonstrated that by measuring the eigenstates shift caused by mode localization, orders of magnitude improvement in sensitivity of mass change was observed [10]. Various groups demonstrated that orders of magnitude enhancement in sensitivity of stiffness change [12–15] and force [16] could be achieved using this approach. Another advantage of this type of device is its intrinsic common mode rejection [17].

The force to be measured can be applied to a resonator in different directions, depending on the application: one way is to apply a vertical force or force gradient to the tip of a horizontal cantilever, as demonstrated in [18]. This approach is widely used for atomic force microscopy (AFM) due to its simplicity. However, for non-contact AFM applications, when the gradient of the Van der Waals force exceeds the stiffness of the cantilever, snap-down

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instability occurs [19], which is analogous to the pull-in effect occurring in parallel plate actuation. Hence, a large stiffness for the vibrating structure is required for some applications, which, in turn, deteriorates the force sensitivity and resolution of the sensor. To reach maximum stability while not compromising the sensitivity, an alternative method is to apply the force along the length of a beam [20,21]. Due to a relatively high longitudinal stiffness of a beam, the instability is alleviated [21].

In this work, a novel proof-of-concept force sensor consisting of three resonators with enhanced force sensitivity is presented. The reason for using a 3DoF resonator system is that, the third resonator located in between two identical resonators reduces the energy propagation due to its absorption of energy, thus increases the energy attenuation along the chain. Consequently, it enhances the mode localization when a structural disorder is present. It has been demonstrated in both theory and by measurement results [22,23] that a 3DoF weakly coupled resonator sensor can exhibit enhanced sensitivity compared to existing 2DoF mode-localized sensors. The device was fabricated using a silicon-on-insulator process, and tested electrically. The external force was a quasi-static electrostatic force applied along the direction of the beam length, which avoided the potential instability mentioned above. The 3DoF force sensor utilized the mode shape change due to a stiffness perturbation introduced by an external force. The vibration amplitude ratio of two resonators at one mode of interest was used to measure the mode shape change. Two orders of magnitude improvement in sensitivity, compared to 1DoF resonator sensors with frequency shift as an output signal, was observed from the measurement. In addition to sensitivity, resolution and dynamic range of the force sensing device is also discussed.

2. Theory

2.1. Force sensing mechanism

To understand the behaviour of the 3DoF resonator force sensor, the system is modelled as a lumped parameter block diagram as shown in Fig. 1a. A schematic drawing of the left resonator to which the force to be measured is applied, is shown in Fig. 1, and a SEM image of our proof-of-concept chip is shown in Fig. 1c.

A tether structure [24] was used in our design to allow the transmission of an axial electrostatic force to the suspension beams of the left resonator. In addition, it is also used to impede the movement of the electrode attached to the bottom of the suspension beams when the resonator is vibrating, so that the electrostatic force is kept as constant as possible. Therefore, the tether was made wide in the x-axis (170 μm), but thin in the y-axis (5 μm) in our design.

The design ensures that the tether has a high mechanical stiffness in the x-direction. In addition, when the displacement of the resonator in the direction of vibration is small compared to the length of the beam, the movement of the resonator in the y-axis is negligible. Consequently, the tether efficiently constraints the movement of the electrode attached to the suspension beams, and thus it can also be regarded as a fixed end for the two suspension beams attached.

In the y-axis, the tether, which is a cantilever beam in essence, has a stiffness of [25]:

$$K_{\text{tether}} = \frac{Etw_t^3}{4L_t^3} \tag{1}$$

where E, t, w_t, L_t are the Young's modulus, the thickness of the device, the width in the y-axis and effective length of tether, respectively. The longitudinal stiffness of the suspension beam is given by [25]:

$$K_{\text{long}} = \frac{Etw}{L} \tag{2}$$

where w and L are the width in the x-axis and the length of the suspension beam.

To applied forces in the negative y-direction, the tether and the suspension beams act similarly to two springs in parallel [24]. Ideally, the tether does not absorb any force applied in the y-axis, so that all the forces can be measured by the resonator. For our design, the shortest effective length of the tether is 60 μm, resulting in a maximum stiffness of $K_{\text{tether}} = 538$ N/m. Whereas in the y-axis, suspension beams 1 and 3 are in series, therefore the effective longitudinal stiffness is $K_{\text{long}} = 2.48 \times 10^4$ N/m. This indicates that more than 97.9% of the force applied is absorbed by the suspension beams, with less than 2.1% of the force exerting on the tether. Hence, we are able to assume that the entire electrostatic force is transmitted to the resonators for measurement.

When two different DC voltages are applied to the resonator and the electrode below, an electrostatic force is generated in the negative y-axis pulling the resonator. Due to the relatively large length of the electrode in the x-axis of 160 μm compared to the air gap of 4.5 μm, the fringe field can be neglected. Assuming small displacements in the y-axis, the tensile force for the resonator T in terms of voltage difference ΔV between the resonator and the electrode, cross-sectional area of electrode A_e, air gap d_e and dielectric constant of vacuum ε_0 is given by [25]:

$$T = \frac{\varepsilon_0 A_e \Delta V^2}{2d_e^2} \tag{3}$$

For an applied force in the y-axis, the two identical suspension beams (beams 3 and 4 in Fig. 1), are in parallel. Hence the tensile force T is evenly distributed to the two suspension beams. Furthermore, the suspension beams 1 and 3 are in series, so are suspension beams 2 and 4. Therefore, the tensile force applied on each suspension beam equals to $T/2$.

The suspension beams have one end fixed, while the other end moves perpendicular with respect to the beam length. Given the displacement functions along the axis of the beam for these boundary conditions [26], the stiffness of each suspension beam under weak axial tensile force $T/2$ is given by [27]:

$$K_{\text{beam}} = \frac{Etw^3}{L^3} + \frac{0.6T}{L} \tag{4}$$

Moreover, due to the high longitudinal stiffness of the suspension beams, the elongation of the beams are trivial compared to the beam length L. For a tensile force of 1 μN, the resulting elongation of the beams is less than 0.1 nm, which is negligible compared to the beam length of 300 μm; the strain change is therefore neglected. The stiffness perturbation introduced by the tensile force, normalized to the effective stiffness of the resonator K, is therefore:

$$\frac{\Delta K_{\text{force}}}{K} = \frac{2.4T}{LK} \tag{5}$$

With the coupling voltage V_c applied, suppose d is the air gap between parallel plates and A, A_{cf} are the cross-sectional area of the actuation parallel plate and the comb finger overlap, respectively. Neglecting the intrinsic tension introduced during fabrication process, the effective stiffness is given by [15]:

$$\begin{aligned} K &= 4 \times K_{\text{beam}} - K_{\text{elec}} \\ &= \frac{4Etw^3}{L^3} - \frac{\varepsilon_0 (A + 6A_{cf}) V_c^2}{d^3} \end{aligned} \tag{6}$$