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Size-dependent electromechanical buckling of functionally graded electrostatic nano-bridges

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ABSTRACT

This study explored the electromechanical buckling (EMB) of beam-type nanoelectromechanical systems (NEMS) by considering the nonlinear geometric effect and intermolecular forces (Casimir force and van der Walls force) based on modified couple stress theory. To model the system, a slender nanobeam made of functionally graded material (FGM) with clamped-guided boundary conditions, which is under compressive or tensile axial loads as well as symmetric and nonlinear electrostatic and intermolecular transverse loads, is used. Considering the Euler–Bernoulli beam theory and using the principle of minimum potential energy and the variational approach, the governing equation as well as the related boundary conditions is derived. To discretize the equation and its related boundary conditions, and to solve the equations, the generalized differential quadrature method (GDQM) is employed. Finally, after validation of the results, the effects of size, length, power law index, and the distance between the two fixed and movable electrodes on the bucking of the system are discussed and examined.

1. Introduction

Electromechanical systems are classified as conventional electromechanical systems, microelectromechanical systems (MEMS), and nanoelectromechanical systems. In recent years, fundamental and theoretical research, manufacturing systems in laboratory and nanoscale, and high efficiency computer have resulted in great achievements in NEMS. The systems are widely used as an actuator mechanism in the high-tech systems such as aerospace, robotics, sensors and monitors. Design and analysis of such systems are not easy, because electromechanically problems are mostly nonlinear and because significant phenomena such as pull-in instability, electromechanical buckling (EMB) and related design need to be investigated [1].

In view of the fact that NEMS are in the nanoscale, the significant phenomena must be investigated in this scale. Here, three significant phenomena have been examined in the nanoscale in the modeling of nano-bridge.

The first significant phenomenon is the EMB which consists of mechanical buckling and electromechanical bifurcation. Mechanical buckling is a well-known phenomenon in the engineering design. A physical phenomenon of a reasonably straight, slender member bending laterally (usually abruptly) from its longitudinal position due to compression is referred to as mechanical buckling. Considerable research has been carried out in this field such as: size dependent buckling analysis based on higher order theories [2,3], thermal and size effect on free vibration and buckling of micro-beams [4], buckling of beams and columns under combined axial and horizontal loading with various axial loading application locations [5] and also buckling analysis of CNTs and nano-beams using nonlocal elasticity theory [6,7].

The electromechanical bifurcation is a kind of instability that results from the inherent nature of electrostatic forces in symmetric actuators. The response of slender structures which are simultaneously under axial stress and electrostatic field, and, as a result, are simultaneously under axial buckling and electromechanical bifurcation, is referred to as EMB which has been discussed less. Electromechanical buckling response was first set theoretically and experimentally by Abu-Salih and Elata [8,9].

It should be mentioned that microfabricated double-clamped beams attached to unmovable anchors, are often pre-loaded by an axial force. This force originates in a residual stress resulting from the fabrication process [10], appearing due to a temperature variation [11–13] or applied intentionally in order to control the stiffness. This pre-load in the beam-type nano-bridges causes initial curved configuration (arches). These curved beams loaded by concentrated or distributed transverse forces may exhibit the existence of two different stable equilibrium under the same loading which is named snap-through instability. The theoretical and

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experimental snap-through instability analysis along pull-in instability or buckling actuated by a distributed deflection dependent electrostatic force was presented in [14–21] for static and dynamic loadings. Recently, a very interesting study has investigated the snap-through buckling of pre-stressed micro beams [22]. In this study, the devices were fabricated from single crystal silicon on insulator wafer using deep reactive ion etching. The in-plane quasi-static beam response was video recorded and analyzed by means of image processing [22].

However, in this paper, the beam structure has a flat initial configuration under an electrostatic loading without initial arch-shape, and constant residual stress is not assumed because of clamped-guided beam model for nano-bridges; therefore, the snap-through buckling has not been investigated.

The second important phenomenon in the NEMS is the consideration of the size effect. As clear from the references, in classical theories, in examining small size structures, the size effect parameters of the material's length is not considered in the constitutive formulation. However, experimental results show that the size effect has a significant role in the static and dynamic behavior of structures and materials in the nanoscale. Hence, today, methods such as molecular dynamic (MD) simulation and experimental results are used to examine and consider the size effect in the nanoscale. However, it should be noted that considering that methods such as MD simulation and laboratory methods are expensive, many researchers use higher order and non-classical theories which contain at least one effect of the parameter of material length scale in comparison with the classical theory. Some of the higher order theories are the nonlocal theory, couple stress theory and strain gradient theory which are used for predicting the dependency on the size effect of micro/nanostructures [23–25]. Yang et al. obtained the modified couple stress theory with simplification of the couple stress theory. A length scale is preferred in this theory to the classic one to investigate the size dependent [24].

The third phenomenon which needs to be taken into account in NEMS is van der Waals (vdW) or Casimir intermolecular force. The vdW force in sizes lower than a few tens of nanometers affects the function of NEMS. This force is a general attraction in nature. It is a short range force, such that if it distances from the plate, it quickly becomes zero. The other phenomenon which is effective in distances more than a few nanometers is the Casimir force. This force is produced by the oscillations of the magnetic force in the vacuum between the two plates. When the distance between the two surfaces is large enough, i.e., bigger than plasma wavelength (for metals) and attraction (for dielectrics) of the material's surface, the virtual photons emitted from the atom of one surface will not reach the second surface during their lifetime. In this case, the interplay between the two surfaces is expressed with the Casimir force. Several research studies have been done in this field such as: size dependent pull-in instability of torsional nano-actuator [25], theoretical study of the Casimir attraction on the pull-in and buckling instabilities [26,27], modeling the dispersion forces on the pull-in instability [28–30], Casimir effect on the pull-in parameters of nanometer switches [31], nonlinear behavior for nanoscales electrostatic actuators with Casimir force [32] and influence of van der Waals and Casimir forces on electrostatic torsional actuators [33].

Because of the growth of mechanical materials and their application in the micro/nanoscale, the use of functionally graded materials (FGMs) is sometimes preferred over the use of materials with fiber structure, particularly under thermal loads, because there is no internal or boundary gap between them. When external force is applied to these materials, stress peaks in the structure of such materials diminish, and, consequently, failure due to lack of internal cohesion and stress concentration is prevented. Therefore, besides the three phenomena mentioned above, the effects of FGMs on the EMB must be examined, too. These materials are formed in two or more phases with continuous variable dispersion. Phase dispersion variation can affect weight or volume fraction, direction, or shape of the object. With the development of FGM technology, these materials are used in MEMS/MEMS and atomic microscopes because of their high and optimal function and sensibility [34]. Studies have been done to examine the use of these materials in the investigation of buckling like mechanical buckling of FGM micro beam based on the couple stress theory [35–37], the strain gradient theory [23] and the nonlocal Timoshenko beam theory [38].

Now, by modeling the EMB of FGM nanobeam in NEMS and taking into consideration the aforementioned phenomena in the nanoscale, the governing equation of the system in nonlinear and its appropriate solution method need to be used, too. For this purpose, this study used the generalized differential quadrature method (GDQM). The DQ method is a numerical discretization technique for the approximation of derivatives. This method is based on the idea of conventional integral quadrature. The key to DQM is to determine the weighting coefficients for the discretization of a derivative of any order. In recent years, the application of the DQM to solve the structural differential equations and the size dependent problems has been of interest to many researchers. This method is developed by the assumption that the partial derivative of a function with respect to a space variable of a given discrete point can be expressed as a weighted linear sum of the function values at all discrete points in the domain of that variable. The application of the DQM covers almost all the areas of structural and vibration analysis of beams, arches, shafts, plates and shells. Some researchers extensively studied the deflection, buckling, and vibration problems using DQM [39]. Also, some research has probed the beam and shell vibration [40–42], pull-in instability in NEMS [43] and buckling analysis of MEMS by DQM [3].

With respect to aforementioned assumptions, the EMB of FGM nano-bridge based on the modified couple stress theory is investigated in this paper with a view to the geometrical nonlinear effect and transverse electrostatic and intermolecular nonlinear forces. A slender FGM nanobeam with clamed-guided boundary condition is used to model the system. Considering the Euler–Bernoulli beam theory and using principle of minimum potential energy, the governing equations and associated boundary conditions are derived. To solve the equation and associated boundary conditions, GDQ method is employed. Finally, after validation of results, the effects of size, length, power law index, and the gap distance between the two fixed and movable electrodes on the bucking of the system are discussed and examined.

2. Basic equations

Based on the modified couple stress theory which is proposed by Yang et al., the strain energy density ($\bar{U}$) and total strain energy (U) of a linear elastic continuum occupying region V are expressed as [24]

$$\bar{U} = \frac{1}{2}\left(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij}\right), \tag{1}$$

$$U = \int_V \bar{U}\mathrm{d}v = \frac{1}{2}\int_V \left(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij}\right)\mathrm{d}V, \tag{2}$$

where σ_{ij}, ε_{ij}, χ_{ij} and m_{ij} are the components of the symmetric part of stress tensor $\boldsymbol{\sigma}$, the strain tensor $\boldsymbol{\varepsilon}$, the derivatoric part of the couple stress tensor $\mathbf{m}$, and the symmetric part of the curvature tensor $\boldsymbol{\chi}$, respectively. Based on classic elasticity theory, the stress–strain relation can be expressed as follows

$$\sigma_{ij} = \lambda tr(\varepsilon)\delta_{ij} + 2\mu\varepsilon_{ij}, \tag{3}$$