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Atmospheric turbulence MTF for optical waves' propagation through anisotropic non-Kolmogorov atmospheric turbulence

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A B S T R A C T

The conventional investigations for atmospheric turbulence have assumed that the refractive-index fluctuations of atmosphere are statistically homogeneous and isotropic. Developments of experimental and theoretical investigations have shown that the isotropic turbulence generally exists near the ground, and in the free atmosphere layer above the ground the anisotropic turbulence appears. Hence, deviations from the previously published results obtained with the isotropic turbulence assumption are possible. In this study, new analytic expressions for the anisotropic atmospheric turbulence modulation transfer function (MTF) are derived for optical plane and spherical waves propagating through anisotropic non-Kolmogorov turbulence. They consider both an anisotropic coefficient and a general spectral power law value in the range 3 to 4. When the anisotropic coefficient equals one (corresponding to the isotropic turbulence), the new results obtained in this work can reduce correctly to the previously published analytic expressions under isotropic non-Kolmogorov turbulence. The derived MTF models physically describe the turbulence anisotropic property of high atmospheric layer. Numerical calculations show that with the increase of anisotropic factor which is proportional to the atmospheric layer altitude, the atmospheric turbulence produces less effect on the imaging system.

1. Introduction

Atmospheric turbulence has a significant degrading impact on the quality of imaging system due to the random fluctuations of atmospheric refractive-index, and this can be described by the MTF. At the very beginning, researches were focused on the Kolmogorov turbulence and analytic expressions for the turbulence MTF were derived. With the development of experimental equipments and theoretical investigations, the atmospheric turbulence has been proved to deviate from prediction of the Kolmogorov model [1–5]. In these cases, the non-Kolmogorov effect is associated with an exponent different from 11/3 (for Kolmogorov turbulence). And a series of non-Kolmogorov atmospheric refractive-index fluctuations spectral models, including the non-Kolmogorov spectrum [6], the generalized exponential spectrum [7] and the generalized modified atmospheric spectrum [8], have been adopted to derive the theoretical expressions for the non-Kolmogorov turbulence MTF [7,9,10]. In these investigations, the isotropic atmospheric turbulence assumption was adopted. The sizes of turbulence eddies were assumed to be the same in both vertical and horizontal directions.

However, laboratory and theoretical results have shown that the atmospheric turbulence can also be anisotropic [11–22] in the free atmospheric above the boundary layer. The horizontal size of these eddies is typically tens of meters across or, in some cases, kilometers across. While the vertical size of the outer scale cells is usually confined to a few meters. In this case, the free atmosphere is highly anisotropic. Recently, many researchers have focused on the investigations of anisotropic non-Kolmogorov turbulence [22–25] which features both the non-Kolmogorov and anisotropic turbulence cases. Toselli [22] used the spectrum introduced by Gurvich [26] and Kon [27] to theoretically investigate the long term beam spread and scintillation index for Gaussian beam under weak anisotropic non-Kolmogorov turbulence with horizontal path. Then, Gudimetla [23,24] studied the log-amplitude correlation function for plane and spherical waves under weak anisotropic non-Kolmogorov turbulence. When turbulence strength continues to increase beyond the weak turbulence regime, Andrews [25] developed mathematical models for the Gaussian beam propagating through weak-to-strong anisotropic non-Kolmogorov turbulence. For the anisotropic non-Kolmogorov atmospheric turbulence MTF, there have been no related investigations so far, and the existing MTF models derived for isotropic non-Kolmogorov turbulence are not applicable.

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In this study, the anisotropic non-Kolmogorov turbulence refractive-index fluctuations spectrum is adopted to investigate theoretically the MTF for optical plane and spherical waves propagating through anisotropic non-Kolmogorov atmospheric turbulence. Numerical calculations are then performed to analyze the impacts of spectral power law values and anisotropic factor on the final expressions.

2. Anisotropic non-Kolmogorov spectrum

The conventional isotropic non-Kolmogorov power spectrum over the inertial subrange takes the form as [6]

$$\Phi_{n\_isotropic}(\kappa,\alpha)=A(\alpha)\cdot\hat{C}_n^2\cdot\kappa^{-\alpha},\quad (0\le\kappa<\infty, 3<\alpha<4). \tag{1}$$

$$A(\alpha)=\frac{1}{4\pi^2}\Gamma(\alpha-1)\cos\left[\frac{\alpha\pi}{2}\right] \tag{2}$$

where, $A(\alpha)$ is a constant which maintains consistency between the refractive index structure function and its power spectrum, α is the general spectral power law value, $\hat{C}_n^2$ is the generalized structure parameter with units $m^{3-\alpha}$, $\Gamma(\cdot)$ is the gamma function. κ is the wavenumber related to the turbulence cell size, $\kappa=\sqrt{\kappa_x^2+\kappa_y^2+\kappa_z^2}$. κ_x, κ_y, and κ_z are the components of κ in the x, y, and z directions.

To introduce the notion of anisotropy into the spectrum model, a more general model of the structure function is adopted, which takes the form as [27]

$$D_n(R,\alpha,\varsigma)=A(\alpha)\hat{C}_n^2\left(\frac{x^2+y^2}{\varsigma^2}+z^2\right)^{\alpha/2},\quad l_0\ll R\ll L_0 \tag{3}$$

In which, R is a vector spatial variable, ς is the anisotropic factor and it carries the burden of representing the anisotropy in this format by assuming different values. ς parameterizes the asymmetry of turbulence cells in both horizontal and vertical directions. When ς equals one, the isotropic turbulence is shown. As the horizontal turbulence outer cell is always bigger than the vertical turbulence cell, the value of ς is always bigger than one. As ς increases, the anisotropic property exhibits more obviously.

Eq. (3) was established to represent the horizontal symmetry usually employed to analyze anisotropic turbulence. By making the changes of variables $x=\varsigma x'$ and $y=\varsigma y'$, the resulting structure function becomes isotropic in the new spatial variable $R'=(x',y',z)$. This is vital in the theoretical investigations of optical waves' propagation through anisotropic turbulence. In view of the relationship between the structure function and the turbulence spectrum $\Phi_n(\cdot)$ [6]:

$$\Phi_n(\kappa,\alpha,\varsigma)=\frac{1}{4\pi^2\kappa^2}\int_0^\infty\frac{\sin(\kappa R)}{\kappa R}\frac{\partial}{\partial R}\left[R^2\frac{\partial D_n(R,\alpha,\varsigma)}{\partial R}\right]dR \tag{4}$$

the resulting $\Phi_n(\cdot)$ will be isotropic in the stretched wave number space $\kappa_x'=\varsigma\kappa_x$, $\kappa_y'=\varsigma\kappa_y$, and $\kappa_z'=\kappa_z$. In this case, $\Phi_n(\cdot)$ for the anisotropic turbulence becomes

$$\Phi_n(\kappa,\alpha,\varsigma)=A(\alpha)\cdot\hat{C}_n^2\cdot\varsigma^2\cdot(\kappa')^{-\alpha}=A(\alpha)\cdot\hat{C}_n^2\cdot\varsigma^2\cdot[\kappa_z^2+\varsigma^2(\kappa_x^2+\kappa_y^2)]^{-\alpha/2} \tag{5}$$

where $\kappa'=\sqrt{\kappa_z^2+\varsigma^2(\kappa_x^2+\kappa_y^2)}$. Eq. (5) catches the essence of anisotropy and is well suited to analytical operations [22,25]. When $\varsigma=1$, the anisotropic non-Kolmogorov spectrum reduces to the conventional isotropic non-Kolmogorov turbulence spectrum. For the analysis directly below, it is assumed that the propagation is in the z direction ($\kappa_z=0$) and the circular symmetry is maintained in the orthogonal xy-plane throughout the path just like [22,25]. Hence, the spectrum model (Eq. (1)) in this case becomes [22,25]

$$\Phi_n(\kappa,\alpha,\varsigma)=A(\alpha)\cdot\hat{C}_n^2\cdot\varsigma^{2-\alpha}\cdot\kappa^{-\alpha},\quad \kappa=\sqrt{\kappa_x^2+\kappa_y^2}. \tag{6}$$

At this time, the relationship between the anisotropic non-Kolmogorov turbulence spectrum and the isotropic non-Kolmogorov turbulence spectrum is established, and the anisotropic property is exhibited by the factor $\varsigma^{2-\alpha}$. This relationship is very useful in the theoretical investigations for optical waves propagating through anisotropic non-Kolmogorov turbulence. In the following analysis, this anisotropic non-Kolmogorov turbulence spectrum will be used to derive the analytic expressions of anisotropic non-Kolmogorov turbulence MTF.

3. The long exposure MTF model for optical waves propagating through weak anisotropic non-Kolmogorov turbulence

Following the analysis of Hufnagel [29] and Fried [30], the long-exposure atmospheric turbulence MTF can be deduced from the formulation of the mutual coherence function (MCF) in the receiver aperture plane, and it is the product of the MCF evaluated at $\rho=\lambda F\nu$. For the case of plane/spherical wave, the long exposure turbulence MTF in the focal plane of the receiver is given by [30]

$$\mathrm{MTF}_{turb}(\nu)=\exp\left[-\frac{1}{2}D_\omega(\lambda F\nu)\right] \tag{7}$$

where ρ is the separation between points in the image plane transverse to the direction, ν is the spatial frequency measured in cycles per unit length, F is focal length. $D_\omega(\lambda F\nu)$ represents the wave structure function for plane/spherical wave, and it is the sum of the log-amplitude structure function and the phase structure function. For the isotropic turbulence, it takes the form as [28]

$$D_{\omega p}(\rho)=8\pi^2k^2\int_0^L dz\int_0^\infty[1-J_0(\kappa\rho)]\Phi_{n\_isotropic}(\kappa,z)\kappa d\kappa \tag{8}$$

$$D_{\omega s}(\rho)=8\pi^2k^2\int_0^L dz\int_0^\infty[1-J_0(\kappa\rho z/L)]\Phi_{n\_isotropic}(\kappa,z)\kappa d\kappa \tag{9}$$

J_0 is the Bessel function of the first kind and zero order, L is the optical path, and $D_{\omega p}(\rho)$ and $D_{\omega s}(\rho)$ represent the plane and spherical wave structure functions, respectively. For the isotropic non-Kolmogorov turbulence, the general spectral power law α will be considered, $\Phi_{n\_isotropic}(\kappa,z)$ will be replaced by $\Phi_{n\_isotropic}(\kappa,z,\alpha)$.

As stated in Section 2, the anisotropic non-Kolmogorov spectrum $\Phi_n(\kappa,\alpha,\varsigma)$ and the isotropic non-Kolmogorov spectrum $\Phi_n(\kappa,\alpha)$ satisfy the relationship $\Phi_n(\kappa,\alpha,\varsigma)=\varsigma^2\cdot\Phi_{n\_isotropic}(\kappa,\alpha)$. As Eqs. (8) and (9) are derived from the Rytov theory, the relationship between the MTF, wave structure function, and the turbulence spectrum keep unchanged regardless of the isotropic or anisotropic turbulence. With the propagation path z considered in the derivations, $\Phi_{n\_isotropic}(\kappa,z)$ in Eqs. (8) and (9) can be replaced directly by $\Phi_n(\kappa,z,\alpha,\varsigma)$. Both the influences of anisotropic factor ς and spectral power law α will be considered.

$$D_{\omega p}(\rho,\alpha,\varsigma)=8\pi^2k^2\int_0^L dz\int_0^\infty[1-J_0(\kappa\rho)]\Phi_n(\kappa,z,\alpha,\varsigma)\kappa d\kappa \tag{10}$$

$$D_{\omega s}(\rho,\alpha,\varsigma)=8\pi^2k^2\int_0^L dz\int_0^\infty[1-J_0(\kappa\rho z/L)]\Phi_n(\kappa,z,\alpha,\varsigma)\kappa d\kappa \tag{11}$$

In the next section, the analytic expressions of plane and spherical wave structure functions for anisotropic non-Kolmogorov turbulence case will be derived, and subsequently the generalized anisotropic turbulence long exposure MTF can be obtained.