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Analytical modeling of curved piezoelectric, Langevin-type, vibrating transducers using transfer matrices

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ABSTRACT

This paper describes a relatively general method based on a transfer matrices approach, for modeling piezoelectric Langevin-type transducers presenting a curved geometry. The obtained equations are simple enough to be solved for a vast number of configurations and be easily implemented in a program. The paper explains how they were obtained and how they can be solved using a method involving transfer matrices. The model was also employed to simulate a real transducer and a good agreement was found between calculated and measured data. Also the results given by model were compared with those obtained with FEM software ANSYS, again the model proved to work well.

1. Introduction

Piezoelectric transducers are currently used in numerous fields such as medical imaging, non-destructive testing, damping control, and energy harvesting [1,2]. Another interesting application is the use of piezoelectric transducers as pure actuators in ultrasonic welding machines, drills or scalpels [3,4], acting as sources of vibrating power. Such devices usually have a similar geometry: there is a back mass and a waveguide that fix the resonance frequencies of the system, pre-stress the ceramics and transfer vibration to the interested region, usually a tip. The modeling of such a device could be quite difficult, wherefore the finite element method is widely used [5,6].

There are of course many, more simple models of curved piezoelectric devices (cantilever beams, laminated plates, membranes containing piezoelectric elements) [7–11] but we could not find among them one which would be suitable to use for a Langevin-type curved transducer, in which we were interested. This could be explained by the fact that most ultrasonic transducers have a straight shape because they are more easy to manufacture and also to study. For such devices there were many studies which implemented 1D models which could take into account the variation of the cross-section, different configurations of piezoelectric materials [12] involving transfer matrices [13] or equivalent electrical transmission lines [14].

These analytical models give only approximate results but can be easily implemented in a simple program and are useful for the design of a piezoelectric vibrating system when there is no FEM software at hand and fast calculations are required. An analytical model for a curved piezoelectric transducer could also be interesting for the same reasons but as we already said we did not manage to find such a model. On the other hand, many studies have been carried out on vibration of curved elastic structures [15–20]. We thus decided to deduce a similar model in which the presence of piezoelectric elements has been taken into account.

The paper is organized as follows. First we describe the used hypothesis and notations, and give the resultant differential equations. Next we explain how these equations can be solved using a transfer matrix approach. We subsequently apply the model to a simple example and also solve it using finite element modeling (**Ansys**). Following this, the two sets of results are compared. In order to validate the model, the prototype of a simple curved transducer was built and in the last part of the paper the measured admittance of piezoelectric ceramics is compared with its calculated counterpart.

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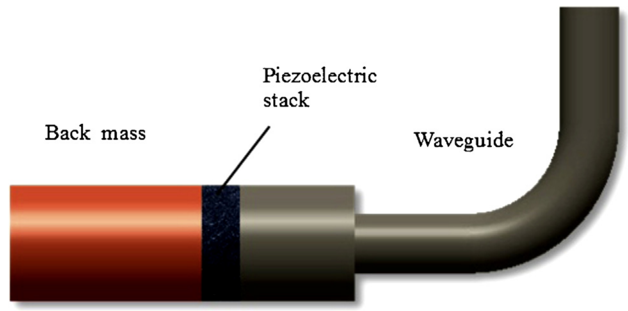

Fig. 1. Possible geometry of a vibrating piezoelectric transducer.

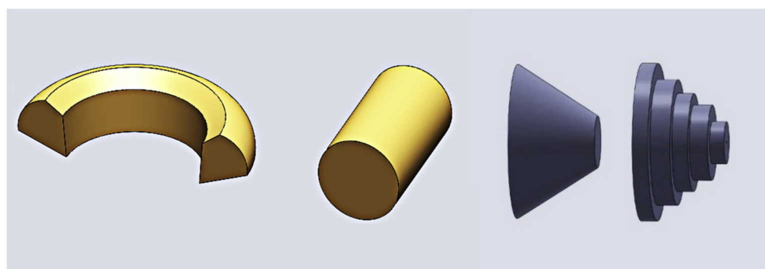

Fig. 2. (Left) Toroid bodies. A cylinder can be considered a toroid with an infinite radius. (Right) Possible discretization of a body with a gradually changing cross-section.

2. Theory

Let us assume that we have a piezoelectric vibrating system, powered by a stack of ceramics excited in longitudinal mode by a sinusoidal voltage. The geometry of this device could resemble the one represented in Fig. 1.

Such a transducer could contain curved parts situated in different planes, and the cross-section could change gradually along the device. On the other hand, the piezoelectric stack is considered to be cylindrical in shape (but its cross-section does not have to be circular) and polarized along its axis. In order to proceed to further analyses, we need to define some basic concepts. First, we should discretize the entire volume, approximating it by a set of successive toroid volumes (Fig. 2).

As will be seen later, this leads to linear differential equations with constant coefficients that are relatively simple to solve. We now numerate each part, starting with the back mass extremity and use the notation $\mathbf{P_n}$ for the $\mathbf{n}$ part. Anywhere inside $\mathbf{P_n}$ we can define a cross-section and we can also find an infinite number of arcs perpendicular to them. Let us from now on consider for each $\mathbf{P_n}$ a particular arc and call it $\mathbf{C_n}$. There should be no obligatory continuity between $\mathbf{C_n}$ and $\mathbf{C_{n+1}}$. For each point of $\mathbf{C_n}$ we denote by l the total length of curves $\mathbf{C_i}$ that connect this point to the one at the extremity of $\mathbf{C_1}$. The point itself is denoted $\mathbf{G}(l)$. Next, for any $\mathbf{G}(l)$ we define a Frenet frame $(G(l), \vec{x}, \vec{y}, \vec{z})$ such that $\vec{z}$ is the tangential vector, $\vec{y}$ is the radial vector and $\vec{x} = \vec{y} \wedge \vec{z}$ (Fig. 3).

For any $\mathbf{G}(l)$, there is a cross section $\mathbf{S}(l)$ and we can suppose for any point $M \in \mathbf{S}(l)$ that its displacement can be expressed as:

$$\vec{u}_M = \vec{u}_G + \vec{\varphi} \wedge \overrightarrow{GM} + \vec{d} \tag{1}$$

Here: $\vec{u}_M$: displacement vector of a point $M \in \mathbf{S}(l)$; $\vec{u}_G$: displacement vector of the point $\mathbf{G}(l)$; $\vec{\varphi}$: rotation vector of the section $\mathbf{S}(l)$; $\vec{d}$: in-plane $\mathbf{S}(l)$ displacement of the point M; $\rightarrow GM$: vector connecting $\mathbf{G}(l)$ to M

$\wedge$: vector product.

Each vector can be decomposed into the corresponding Frenet frame:

$$\vec{u}_G = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \quad \vec{\varphi} = \begin{pmatrix} \varphi \\ \psi \\ \theta \end{pmatrix} \quad \vec{d} = \begin{pmatrix} d_1 \\ d_2 \\ 0 \end{pmatrix}$$

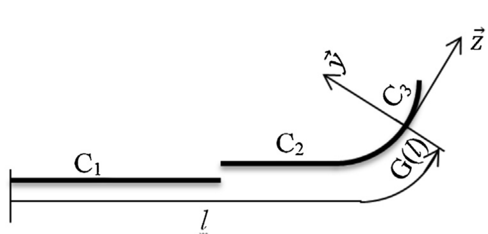

Fig. 3. Schematic view of different defined objects.