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Deterministic and band-limited stochastic energy harvesting from uniaxial excitation of a multilayer piezoelectric stack

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ABSTRACT

Deterministic and band-limited stochastic energy harvesting scenarios using a multilayer piezoelectric stack configuration are investigated for uniaxial dynamic pressure loading. The motivation for exploring this off-resonant energy harvesting problem derives from typical civil infrastructure systems subjected to dynamic compressive forces in deterministic or stochastic forms due to vehicular or human loads, among other examples of compressive loading. Modeling of vibrational energy harvesters in the existing literature has been mostly focused on deterministic forms of mechanical vibration as in the typical case of harmonic excitation, while the efforts on stochastic energy harvesting have thus far considered second-order systems such as piezoelectric cantilevers. In this paper, we present electromechanical modeling, analytical and numerical solutions, and experimental validations of piezoelectric energy harvesting from harmonic, periodic, and band-limited stochastic excitation of a multilayer piezoelectric stack under axial compressive loading in the off-resonant low-frequency range. The deterministic problem employs the voltage output-to-pressure input frequency response function of the harvester for a given electrical load, which is also extended to periodic excitation. The analytical stochastic electromechanical solution employs the power spectral density of band-limited stochastic excitation to predict the expected value of the power output. The first one of the two numerical solution methods uses the Fourier series representation of the excitation history to solve the resulting ordinary differential equation, while the second method employs an Euler–Maruyama scheme to directly solve the governing electromechanical stochastic differential equation. The electromechanical models are validated through several experiments for a multilayer PZT-5H stack under harmonic and band-limited stochastic excitations at different pressure levels. The figure of merit is also extracted for this particular energy harvesting problem to choose the optimal material. Soft piezoelectric ceramics (e.g. PZT-5H and PZT-5A) offer larger power output as compared to hard ceramics (e.g. PZT-8), and likewise, soft single crystals (e.g. PMN-PT and PMN-PZT) produce larger power as compared to their hard counterparts (e.g. PMN-PZT-Mn); and furthermore, single crystals (e.g. PMN-PT and PMN-PZT) generate more power than standard ceramics (e.g. PZT-5H and PZT-5A) for low-frequency, off-resonant excitation of piezoelectric stacks.

1. Introduction

Vibration-based energy harvesting for low-power electricity generation has received growing attention over the last decade [1–4]. The motivation in this research field is due to the reduced power requirement of small electronic components, such as wireless sensor networks used in passive and active monitoring applications. By means of harvesting ambient energy in next-generation wireless electronic systems, it is aimed to minimize the maintenance costs for periodic battery replacement/charging as well as the chemical waste of conventional batteries. Over the last decade, numerous research groups have reported their work on modeling and applications of vibration/kinetic energy harvesting using electromagnetic [5–10], electrostatic [11–13], piezoelectric [14–20], magnetostrictive [21,22], and electrostrictive [23,24] conversion mechanisms, as well as electronic and ionic electroactive polymers [25,26]. In particular, due to the high power density and ease of application of piezoelectric materials in various configurations from meso-scale [15,17] to micro-scale [27–30], piezoelectric energy harvesting has received the greatest attention [3,31–34].

The existing literature of piezoelectric energy harvesting has mostly explored cantilevers whereas very limited work has focused

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on piezoelectric stacks [35–37]. Goldfarb and Jones [38] analyzed a piezoelectric stack under harmonic mechanical load using a lumped parameter model and derived an expression of power efficiency. Feenstra et al. [39] designed an energy harvester that employed a mechanically amplified piezoelectric stack and converted the dynamic tension in the backpack strap to electrical energy. As an alternative to conventional flex-tensional piezoelectric transducer [40], Li et al. [41] designed a flex-compressive mode transducer that used two piezoelectric stacks and two bow-shaped elastic plates for energy harvesting.

In terms of stochastic (random) vibrational energy harvesting, the entire literature of energy harvesting has focused on second-order (resonating) linear and nonlinear configurations. Stochastic vibration energy harvesting models for standard second-order linear energy harvesters under broadband excitation were given by Halvorsen [42] and Adhikari et al. [43]. Stochastic analysis using distributed-parameter piezoelectric energy harvester models including higher vibration modes was presented by Zhao and Erturk [44]. In addition to stochastic energy harvesting with linear stiffness [42–44], researchers have also explored stochastic excitation of second-order monostable [45,46] and bistable [47–53] nonlinear energy harvesters of Duffing oscillator type. However, stochastic excitation of first-order energy harvesters (such as a piezoelectric stack configuration excited within typical ambient energy frequency spectrum) by dynamic pressure loading has not been addressed to date. Typically the fundamental resonance frequency of a stack-type piezoelectric energy harvester under uniaxial loading (one the order of tens of kHz) is much higher than arguably all practical ambient excitation frequency spectra, resulting in first-order dynamic behavior.

The present work investigates deterministic and stochastic energy harvesting scenarios using a multilayer piezoelectric stack under dynamic pressure loading. The motivation for exploring this off-resonant energy harvesting problem derives from typical civil infrastructure systems subjected to dynamic compressive forces in deterministic or band-limited stochastic forms due to vehicular or human loads, among other examples of compressive loading in low-frequency region of piezoelectric stacks. In the following, first, an electromechanical model is given along with frequency response derivations for the modeling of harmonic and periodic excitation cases. The figure of merit to choose the optimal piezoelectric material is also extracted. After that, both analytical and numerical solutions of power generation from band-limited stochastic excitation are summarized. Finally, experimental results are presented to validate the analytical and numerical predictions of low-power electricity generation from harmonic and stochastic uniaxial loading.

2. Deterministic excitation and electromechanical response

2.1. Governing electromechanical equation

Fig. 1 shows a multilayer piezoelectric stack energy harvester configuration under the excitation of dynamic pressure $p(t)$ in the axial direction (3-direction). The formulation given in the following assumes that the highest frequency content of the axial pressure is much lower than the fundamental resonance frequency of the stack so that one-way coupling and first-order behavior can describe the electromechanical system dynamics.[1] If the total of N thickness-poled layers in the stack are connected in parallel to a resistive electrical load (R_l), then the governing circuit equation is obtained from

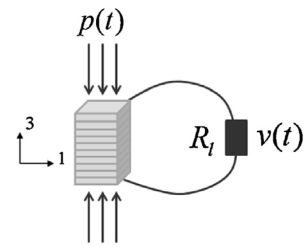

Fig. 1. Schematic of a multilayer piezoelectric stack used for harvesting energy from uniaxial dynamic pressure excitation at off-resonant low frequencies (thickness-poled piezoelectric layers are combined in parallel).

$$\sum_{i=1}^{N} \frac{d}{dt}\left(\int_{A_i} \mathbf{D}\cdot\mathbf{n}dA_i\right) = \frac{v(t)}{R_l}, \tag{1}$$

where $v(t)$ is the voltage response across the load, $\mathbf{D}$ is the vector of electric displacements, $\mathbf{n}$ is the vector of surface normal of the electrodes, and the integration of their inner product is performed over the electrode area A_i of the ith layer. The electric displacement components are obtained from

$$D_i = d_{ijk}T_{A1} + \varepsilon_{ij}^{T}E_j, \tag{2}$$

where T_{jk} and E_j are the stress and electric field tensors, respectively, d_{ijk} is the tensor of piezoelectric strain constants, and ε_{ij}^{T} is the tensor of permittivity constants at constant stress. If contracted index notation (i.e. Voigt's notation: $11\to1, 22\to2, 33\to3, 23\to4, 13\to5, 12\to6$) is used in Eq. (1) along with the electrode and mechanical boundary conditions, the surviving stress ($T_3 = p(t)$) and electric field ($E_3 = -v/h$, where h is the thickness of each piezoelectric layer) components yield the governing electromechanical equation

$$C_p^{eq}\dot{v}(t) + \frac{1}{R_l}v(t) = d_{33}^{eff}A\dot{p}(t), \tag{3}$$

where C_p^{eq} is the equivalent capacitance and d_{33}^{eff} is the effective piezoelectric constant, A is the cross-sectional area on which the pressure is acting (thus $p(t)A$ is the dynamic force transmitted to the stack), and an over-dot represents differentiation with respect to time. For a stack made of N identical layers, the effective piezoelectric strain constant and equivalent capacitance are $d_{33}^{eff} = N\mu d_{33}$ and $C_p^{eq} = N\lambda\varepsilon_{33}^{T}A/h$, where μ and λ are empirical constants that account for the difference between the bulk and thin-layer piezoelectric and dielectric constants as well as the fabrication effects on the stack (μ and λ are close to unity). The dielectric loss of is assumed to be negligible (although it can be taken into account by assuming a complex permittivity constant). Once again, the highest frequency content ($\bar{\omega}$) of the dynamic pressure is assumed to be much lower than the fundamental natural frequency of the stack (ω_n), i.e. $\bar{\omega} \ll \omega_n$; therefore the stress field is insensitive to changing electrical load resistance, making it possible to represent the system dynamics by a single first-order equation unlike the resonant energy harvesting problem [3,15,17].

2.2. Electromechanical frequency response

If the dynamic pressure uniformly acting on the stack shown in Fig. 1 is harmonic of the form $p(t) = p_0e^{j\omega t}$ (where p_0 is the amplitude of the axial pressure, ω is the frequency, and j is the unit imaginary number), then the steady-state voltage response

[1] In other words, the stack is excited at its off-resonant, quasi-static frequencies. This is a realistic assumption for energy harvesting at frequencies below kHz regime using a stack of fundamental resonance frequency on the order of tens of kHz.