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Experimental study of the fractional Fourier transform for a hollow Gaussian beam

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ABSTRACT

Hollow Gaussian beam (HGB) was introduced in [Cai et al., Optics Letters 2003;28:1084–1086 [12]] and the fractional Fourier transform (FRT) for a HGB was studied theoretically in [Zheng, Physics Letters A 2006;355:156–161 [53]] . In this paper, we derive the analytical formula for the truncated FRT for a HGB, and we report experimental observation of the FRT and the truncated FRT for a HGB. The influences of the fractional order and the truncation parameter on the intensity distribution of the HGB in the FRT plane have been studied in detail both theoretically and experimentally. It is found that the FRT optical system provides an efficient way for modulating the beam profile of a HGB. Our experimental results agree well with the theoretical predictions.

1. Introduction

In the past decades, dark hollow beam (DHB) has been investigated extensively and it has found wide applications in free-space optical communications, laser optics, particles trapping, medical sciences, atomic and binary optics [1–40]. Several theoretical models have been proposed to describe various DHBs [9–18]. The conventional DHBs such as Bessel Gaussian beam [9] and TEM_{01}^{*} beam (also known as doughnut beam) [10] usually have a spiral phase, and their dark hollow beam profiles remain invariant on propagation although their beam spots spread. Hollow Gaussian beam (HGB) is one kind of DHB without a spiral phase, and was introduced by Cai et al. in 2003 [12], and it is shown that it has unique propagation properties, i.e., its dark beam profile varies on propagation in free space and its dark hollow beam profile disappears totally in the far field, which are much different from those of the conventional DHBs with a spiral phase. Theoretical model named hollow elliptical Gaussian beam was proposed to describe a DHB of elliptical system without spiral phase [18]. Deng et al. derived the expression for the M^2-factor of a HGB [19]. The focusing properties of a HGB were explored in [20–23]. Paraxial propagation properties of a HGB through aligned or misaligned optical system with truncation were reported in [24,25]. Zhou et al. investigated the nonparaxial propagation properties and the vectorial structure of a HGB [26–30]. Wang et al. carried out investigation of atomic trapping and guiding by a HGB, and found that a HGB is useful for atomic trapping and guiding [31]. Zhao et al. studied the radiation forces on a dielectric sphere produced by highly focused HGB, and found that a focused HGB can be used to trap particles with refractive index larger or smaller than the ambient under suitable conditions [32]. Qiao et al. studied the scintillation index and bit error rate of a HGB in atmospheric turbulence, and they found that a HGB has advantage over a Gaussian beam for reducing turbulence-induced scintillation, which is useful for free-space optical communications [33]. Yadav and Kandpal explored the spectral anomalies of polychromatic HGB and discussed its applications in free-space optical communications [34]. In [35], Philip and Viswanathan found that HGB is useful for generating tunable chain of three-dimensional optical beams. More recently, Wang et al. reported experimental generation of a Laguerre–Gaussian Schell-model beam with the help of a HGB, which is useful for beam shaping [36]. Up to now, several methods have been proposed to generate a HGB [37–39]. Just recently, we proposed a new method to generate a HGB through transforming a Laguerre–Gaussian beam into a HGB [40].

On the other hand, since the concept of fractional Fourier transform (FRT) was first introduced into optics by Ozaktas, Mendlovic and Lohmann in 1993 [41–43], numerous attention has been paid to the FRT and it has found important applications in image encryption, signal processing, beam shaping and beam analysis [44]. In the past several years, the propagation properties of various laser beams through the FRT optical system have been studied extensively, and it is shown that the FRT optical system provides a convenient way to control the statistical properties of laser beams [45–53]. The FRT for a HGB was studied theoretically in [53], and it is found that the intensity distribution of the HGB in

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the FRT plane can be modulated by varying the parameter of the FRT optical system. In this paper, we report experimental study of the FRT and the truncated FRT for a HGB, and explore the influences of the fractional order and the truncation parameter of the FRT optical system on the intensity distribution of the HGB in the FRT plane both theoretically and experimentally.

2. Theory

In this section, we first review briefly the FRT for a HGB, and then we investigate the truncated FRT for a HGB theoretically.

The electric field of a HGB at the source plane is defined as [12]

$$E(r,\theta)=\left(\frac{r^2}{\omega_0^2}\right)^n \exp\left(-\frac{r^2}{\omega_0^2}\right), \tag{1}$$

where r, θ are the radial and azimuthal (angle) coordinates and n is the order of the HGB, ω_0 denotes the source beam waist size of a fundamental Gaussian beam. When $n=0$, Eq. (1) reduces to the electric field of a fundamental Gaussian beam. The area of the dark region increases with the increase of the beam order n (see Fig. 1).

The propagation of a laser beam through a paraxial ABCD optical system can be treated by the following Collins formula [54]:

$$E(\rho,\varphi)=\frac{ik}{2\pi B}\int_0^\infty\int_0^{2\pi} E(r,\theta)\exp\left[-\frac{ik}{2B}(Ar^2-2r\rho\cos(\phi-\theta)+D\rho^2)\right]rdrd\,\theta, \tag{2}$$

where $k=2\pi/\lambda$ is the wavenumber with λ being the wavelength of the beam. A, B, C, D are the elements of the transfer matrix of the paraxial optical system. Substituting Eq. (1) into Eq. (2), we obtain the following analytical propagation formula for a HGB passing through a paraxial ABCD optical system [12]:

$$E(\rho,\varphi)=\frac{ikAn!}{2B\omega_0^{2n}}\left(\frac{1}{\omega_0^2}+\frac{ikA}{2B}\right)^{-n-1} \exp\left[-\frac{ikDr^2}{2B}-\frac{(kr/2B)^2}{(1/\omega_0^2)+(ikA/2B)}\right]L_n^0\left[\frac{(kr/2B)^2}{(1/\omega_0^2)+(ikA/2B)}\right], \tag{3}$$

where L_n^0 denotes the Laguerre polynomial of order n and 0.

According to [41], the optical system for implementing the FRT for an optical beam is given as shown in Fig. 2. $E(r,\theta)$ and $E(\rho,\phi)$ denote the electric field of the beam in the input plane and the FRT plane, respectively. A thin lens with focal length $f/\sin\phi$ is located between the input pane and the FRT plane, and both the distance from the input plane to the thin lens and the distance from the thin lens to the FRT plane equal to $f\tan(\phi/2)$. Here f is the standard focal length, and $\varphi=p\pi/2$ with p being the fractional order of the FRT optical system. When $p=4n+1$ with n being any integer, the FRT optical system reduces to the conventional Fourier transform optical system. The transfer matrix of the FRT optical system in Fig. 2 is given as

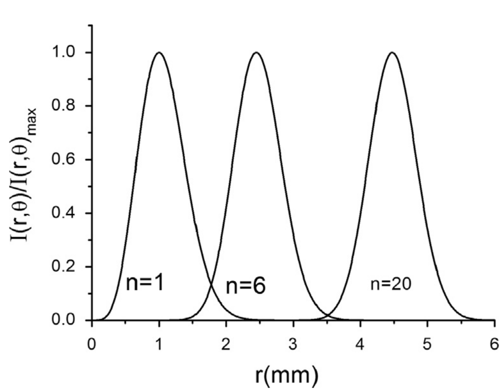

Fig. 1. Normalized intensity distribution of a HGB for different values of n with $\omega_0=1$ mm.

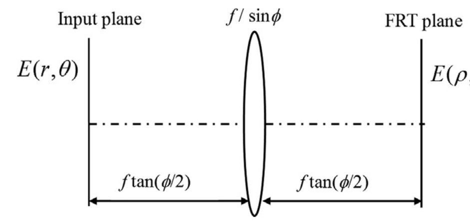

Fig. 2. Optical system for performing the FRT for an optical beam.

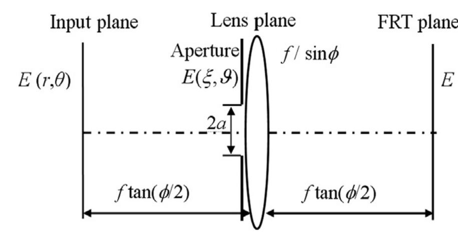

Fig. 3. Optical system for performing the truncated FRT for an optical beam.

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}=\begin{pmatrix} 1 & f\tan(\phi/2) \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\sin\phi/f & 1 \end{pmatrix}\begin{pmatrix} 1 & f\tan(\phi/2) \\ 0 & 1 \end{pmatrix} =\begin{pmatrix} \cos\phi & f\sin\phi \\ -\sin\phi/f & \cos\phi \end{pmatrix}. \tag{4}$$

Applying Eqs. (3) and (4), one can study the properties of a HGB in the FRT plane numerically. Numerical results in [53] have shown that the FRT optical system can be used to modulate the intensity distribution of a HGB by varying the fractional order.

Now we study the truncated FRT for a HGB. In a practical case, most optical systems contain some aperture confinement (i.e., truncation), thus it is interesting to study the influence of truncation on the properties of a HGB in the FRT plane. Fig. 3 shows the optical system for performing the truncated FRT for an optical beam, which is similar to Fig. 2, except that a circular aperture with radius a is located just before the thin lens in Fig. 3. The whole optical system can be divided into two sections: The first section is the free-space propagation of the beam from the input plane to the truncated lens plane, and the second section is also the free-space propagation of the beam from the truncated lens plane to the FRT plane.

The transfer matrix of the free space between the input plane and the truncated lens plane reads as

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}=\begin{pmatrix} 1 & f\tan(\phi/2) \\ 0 & 1 \end{pmatrix}. \tag{5}$$

Applying Eqs. (3) and (5), we obtain the expression for the electric field of a HGB in the truncated lens plane as follows:

$$E(\xi,\vartheta)=\frac{ikn!p_1^{-(n+1)}}{2\omega_0^{2n}f\tan(\phi/2)}\exp\left[-\frac{ik}{2f\tan(\phi/2)}\xi^2-\frac{k^2\xi^2}{4f^2\tan^2(\phi/2)p_1}\right] \times L_n^0\left[\frac{k^2\xi^2}{4f^2\tan^2(\phi/2)p_1}\right], \tag{6}$$

where

$$p_1=\frac{1}{\omega_0^2}+\frac{ik}{2f\tan(\phi/2)}.$$

The propagation of the HGB from the truncated lens plane to the FRT plane can be studied by the following extended Collins