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Analytical solution for an anomalous hollow beam in a fractional Fourier transforming optical system with a hard aperture

Kuilong Wang [a], Chengliang Zhao [b,*]

[a] Department of Physics, Hangzhou Normal University, Hangzhou 310036, China
[b] School of Physics Science and Technology, Soochow University, Suzhou 215006, China

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ABSTRACT

Based on the fact that a hard aperture function can be expanded into a finite sum of complex Gaussian functions, the approximate analytical expressions for the output field distribution of an anomalous hollow beam (AHB) passing through an apertured fractional Fourier transform (FRT) system are derived. By using the approximate analytical formulae and diffraction integral formulae, the propagation properties of an AHB in circular and rectangular apertured FRT system are studied numerically. The results show that this method provides a convenient tool for studying the propagation properties of an AHB passing through apertured FRT system, and the apertured FRT system can be applied to laser beam shaping conveniently.

1. Introduction

The Fractional Fourier transform (FRT) was first introduced into optics by Ozaktas, Mendlovic and Lohmann in 1993 [1–3], since then it has been extensively studied due to its important applications in signal processing, optical image encryption, beam shaping, and beam analysis [4–9]. The propagation and transformation of various configurations of laser beams, such as elliptical Gaussian, flattened Gaussian, dark hollow, and partially coherent beams through FRT optical system have been studied in detail [10–16]. It is shown that the FRT optical system can be applied to control the laser beam properties, such as intensity, spectrum, degree of coherence and polarization.

On the other hand, the beams with zero-central intensity have been widely investigated both theoretically and experimentally due to their potential applications in guiding and manipulating the neutral atoms and micro-sized particles [17–20], such as the LG modes, dark hollow beams, Bessel–Gaussian beams, higher-order Mathieu beams and hollow Gaussian beams. Recently, Wu et al. observed in an experiment an anomalous hollow beam (AHB) of elliptical symmetry with an elliptical solid core, which can be used for studying the transverse instability and provide a powerful tool for studying the linear and nonlinear particle dynamics in the storage ring [21]. More recently, Cai first proposed two convenient theoretical models to describe AHB [22,23]. The propagation properties of AHB in free space, in turbulent atmosphere, in misaligned optical system, in fractional Fourier transform, and the coherent and partially coherent AHBs have been studied [22,24–27].

Generally, the FRT system without aperture has been extensively studied and used. However, in practical case, the apertures always exist in most optical systems, for example, the finite size of lens. So it is necessary and of practical significance to study the propagation of a laser beam through an apertured optical system. It has been found that the aperture in optical systems has significant influence on the performance of the system. The apertured optical system also has significant applications in speckle metrology [28–30]. Propagations of some laser beams such as flattened Gaussian, Hermite–Gaussian, Laguerre–Gaussian, elliptical Gaussian, and flat-topped multi-Gaussian beams through apertured FRT optical system have been studied [31–36]. To the best of our knowledge, the propagation of AHB through apertured FRT system has not been studied. This paper is devoted to studying the propagation of an AHB through apertured FRT system based on the expansion of a hard aperture function into a finite sum of complex Gaussian functions. Properties of an AHB in the FRT plane after passing through apertured FRT system with circular or rectangular aperture are studied numerically.

2. Theory

The optical system for performing apertured FRT system is shown in Fig. 1. An aperture is located just before the thin lens with focal length $f = f_s/\sin\phi$. f_s is the standard focal length

* Corresponding author. Tel.: +86 15190081996; fax: +86 051269153532.
E-mail addresses: chengliang_zhao@yahoo.com.cn, zhaochengliang@suda.edu.cn (C. Zhao).

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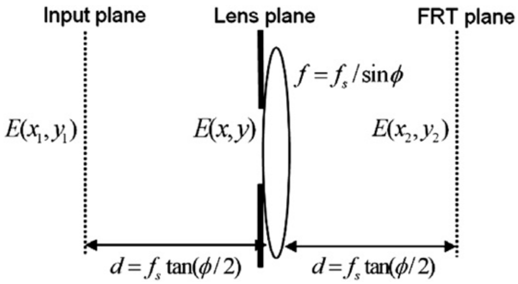

Fig. 1. Optical systems for performing the apertured FRT system.

and $f=p\pi/2$ with p being the fractional order of the FRT optical system. The whole optical system can be divided into two sections: The first section is the free-space propagation of the AHB from the input plane to the apertured lens plane, and the second section is the propagation of the AHB from the apertured lens plane to the FRT plane, the transfer matrices of the two sections are described by $\{A_1, B_1, C_1, D_1\}$,$\{A_2, B_2, C_2, D_2\}$, respectively. And the matrices are given by

$$\begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}, \tag{1}$$

$$\begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} = \begin{pmatrix} 1-\frac{d}{f} & d \\ -\frac{1}{f} & 1 \end{pmatrix}. \tag{2}$$

Within the framework of paraxial approximation, the propagation of a laser beam passing through the *ABCD* optical system can be treated by Collins diffraction integral formula [37] (it should be pointed out that the Collins integral is often called a Linear Canonical Transform (LCT), the Collins integral is the special case of the LCT), for the first one we have

$$E(x,y) = \frac{i}{\lambda B_1}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E(x_1,y_1) \times\exp\left\{-\frac{ik}{2B_1}\left[A_1(x_1^2+y_1^2)-2(x_1x+y_1y)+D_1(x^2+y^2)\right]\right\}dx_1\,dy_1, \tag{3}$$

and for the second one we have

$$E(x_2,y_2) = \frac{i}{\lambda B_2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K(x,y)E(x,y) \times\exp\left\{-\frac{ik}{2B_2}\left[A_2(x^2+y^2)-2(xx_2+yy_2)+D_2(x_2^2+y_2^2)\right]\right\}dx\,dy, \tag{4}$$

where $E(x_1, y_1)$, $E(x, y)$ and $E(x_2, y_2)$ are the electric fields of the laser beam at the input plane, the apertured lens plane and the output plane, respectively. $K(x, y)$ is the hard aperture function, $k=2\pi/\lambda$ is the wave number, λ is the wavelength. And the constant phase terms in Eqs. (3) and (4) have been omitted which have no influence on the output intensity distribution. The A_1, B_1, C_1 and D_1 are the elements of the matrix of the first section of the optical system, and the A_2, B_2, C_2 and D_2 are the elements of the matrix of the second section of the optical system, they are expressed as Eqs. (1) and (2).

We assume that the laser beam in the input plane is an AHB, whose electric field can be described as superposition of astigmatic Gaussian modes and astigmatic doughnut modes as follows [22]:

$$E(x_1,y_1) = E_0\left(-2+\frac{8x_1^2}{w_{0x}^2}+\frac{8y_1^2}{w_{0y}^2}\right)\exp\left(-\frac{x_1^2}{w_{0x}^2}-\frac{y_1^2}{w_{0y}^2}\right) \tag{5}$$

where E_0 is a normalized constant, w_{0x} and w_{0y} are the beam waist sizes of an astigmatic Gaussian beam in x and y directions, respectively.

If the aperture before the thin lens is circular and its radius is a, $K(x,y)$ can be repressed as

$$K(x,y) = \begin{cases} 1, & x^2+y^2 \le a^2 \\ 0, & x^2+y^2 > a^2 \end{cases} \tag{6}$$

Then the hard aperture function can be expanded as the following sum of complex Gaussian functions [38–40]:

$$K(x,y) = \sum_{n=1}^{N} U_n \exp\left[-\frac{V_n}{a^2}(x^2+y^2)\right], \tag{7}$$

where U_n and V_n are the expansion and Gaussian coefficients, respectively, which could be obtained by optimization-computation directly, a table of U_n and V_n can be found in [38,41]. This expansion method has been proved reliable and efficient. The simulation accuracy improves as N increases. For a hard aperture, $N=10$ assures a very good agreement with the straightforward diffraction integration [38,41], so we take $N=10$ in the following numerical calculations.

Substituting Eqs. (3), (5) and (7) into (4) and using the following integral formulae:

$$\int_{-\infty}^{+\infty} \exp(-\delta x^2)\,dx = \sqrt{\pi/\delta}, \tag{8a}$$

$$\int_{-\infty}^{+\infty} x\exp(-\delta x^2)\,dx = 0, \tag{8b}$$

$$\int_{-\infty}^{+\infty} x^2\exp(-\delta x^2)\,dx = \frac{1}{2\delta}\sqrt{\pi/\delta}, \tag{8c}$$

after tedious but straightforward integration, we can obtain the following approximate analytical expression of the output field distribution of an AHB passing through a circular apertured FRT system:

$$E(x_2,y_2) = \frac{k^2 w_{0x}w_{0y}E_0}{d[(2d+ikw_{0x}^2)(2d+ikw_{0y}^2)]^{5/2}}\exp\left[-\frac{ik}{2d}(x_2^2+y_2^2)\right] \times\sum_{n=1}^{N}\frac{U_n}{\sqrt{G_nH_n}}\left(\alpha+\frac{2\beta}{G_n}+\frac{2\gamma}{H_n}-\frac{k^2\beta}{d^2G_n^2}x_2^2-\frac{k^2\gamma}{d^2H_n^2}y_2^2\right) \times\exp\left[-\frac{k^2}{4d^2}\left(\frac{x_2^2}{G_n}+\frac{y_2^2}{H_n}\right)\right], \tag{9}$$

where

$$\alpha = -48d^4-32ikd^3(w_{0x}^2+w_{0y}^2)+k^4w_{0x}^4w_{0y}^4 +4k^2d^2(w_{0x}^4+4w_{0x}^2w_{0y}^2+w_{0y}^4), \tag{10}$$

$$\beta = k^2w_{0x}^2(2d+ikw_{0y}^2)^2, \tag{11}$$

$$\gamma = k^2w_{0y}^2(2d+ikw_{0x}^2)^2, \tag{12}$$

$$G_n = \frac{V_n}{a^2}+\frac{ik\cos\phi}{2d}+\frac{ik}{2d+ikw_{0x}^2}, \tag{13}$$

$$H_n = \frac{V_n}{a^2}+\frac{ik\cos\phi}{2d}+\frac{ik}{2d+ikw_{0y}^2}. \tag{14}$$

According to Carter [42], the effective beam size of an AHB in the FRT plane is defined as follows:

$$W_s = \sqrt{\frac{2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s^2|E(x_2,y_2,z_2)|^2\,dx_2\,dy_2}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|E(x_2,y_2,z_2)|^2\,dx_2\,dy_2}} \quad (s=x_2,y_2), \tag{15}$$

where W_x and W_y are the effective beam sizes of an AHB in the x and y directions, respectively. Substituting Eq. (9) into Eq. (15), we obtain (after some operation) the expressions for the effective beam size W_x of an AHB in the FRT plane after passing through