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The far-field divergent properties of an Airy beam

Yiqing Xu, Guoquan Zhou *

School of Sciences, Zhejiang A & F University, Lin'an 311300, Zhejiang Province, China

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ABSTRACT

Based on the method of the stationary phase, the analytical expression of the far-field of an Airy beam has been derived. According to the obtained electromagnetic representations, the formulae of the energy flux and the power of an Airy beam are presented in the far-field. The analytical formulae of the far-field divergence angles that are defined by the second-order moment of the energy flux are also derived. The energy flux distribution of an Airy beam is depicted in the far-field. The power in the far-field and the far-field divergence angles depend on the transverse scale and the modulation parameter. The influences of the transverse scale and the modulation parameter on the power in the far-field and the far-field divergence are discussed in detail.

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1. Introduction

An Airy beam is the solution to the force-free Schrödinger equation. An Airy beam exhibits a nonspreading property in vacuum [1] and has the characteristic of freely accelerating [2,3]. The Airy beam can be generated using the phase-only patterns [4–6] or using the three-wave mixing processes in asymmetric nonlinear photonic crystals [7]. The properties including self-healing, the evolution of the Poynting vector and angular momentum, the phase behavior, and the beam propagation factor of an Airy beam have been widely investigated [8–11]. In order to explain the intriguing features of an Airy beam, some methods such as the method of geometrical optics and the method of the Wigner distribution function have been proposed [12,13]. The propagation of an Airy beam through optical systems described by the *ABCD* matrices with complex elements has been examined [14]. Nonparaxial diffraction of the Airy beams has been analyzed by solving the exact vectorial Helmholtz equation using boundary conditions at a diffraction aperture [15]. The propagation of an Airy beam in water [16], in a nonlinear medium [17,18], and in turbulence [19,20] has been also investigated. Airy beams have crucial applications in optical micromanipulation [21–23].

Many researches and applications are involved in the far-field divergent properties of optical beams. Therefore, the far-field divergence angles, which are defined by the second-order moment of the energy flux, of an Airy beam are to be investigated in the remainder of this paper. The far-field divergence angle indicates the divergence of an optical beam and is one of the important parameters to describe the beam quality. To present the far-field divergence angles, we should derive the far-field of an Airy beam using the method of stationary phase [24,25].

2. The far-field divergence angles of an Airy beam

The Airy beam is treated to be polarized in the *x*-axis and propagates toward the half free space $z\geq 0$. The *z*-axis is taken to be the propagating axis. The initial transverse electric field of the Airy beam in the source plane $z=0$ is described as [2,3]

$$\begin{bmatrix} E_x(x,y,0) \\ E_y(x,y,0) \end{bmatrix} = \begin{bmatrix} Ai\left(\frac{x}{w_0}\right)Ai\left(\frac{y}{w_0}\right)\exp\left(\frac{ax}{w_0}+\frac{ay}{w_0}\right) \\ 0 \end{bmatrix}, \tag{1}$$

where w_0 is the transverse scale. $Ai(\cdot)$ is the Airy function, and a is the modulation parameter. Fig. 1 shows the contour graph of the normalized intensity distribution of an Airy beam in the source plane. In Fig. 1(a), $a=0.1$ and $w_0=0.2\lambda$ with λ being the optical wavelength. $a=1$ and $w_0=\lambda$ in Fig. 1(b). The modulation parameter a determines the number of lateral side lobes. With the increasing value of a, the number of lateral side lobes decreases until all the lateral side lobes disappear.

By means of the method of vector angular spectrum, the propagating electric field of an Airy beam turns out to be

$$\boldsymbol{E}(x,y,z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}A_x(p,q)\left(\boldsymbol{e}_x-\frac{p}{\gamma}\boldsymbol{e}_z\right)\exp\left[ik(px+qy+\gamma z)\right]dpdq, \tag{2}$$

where $\boldsymbol{e}_x$ and $\boldsymbol{e}_z$ are the two unit vectors in the *x*- and *z*-directions, respectively. $\gamma=(1-p^2-q^2)^{1/2}$, and $k=2\pi/\lambda$ is the wave number. $A_{\boldsymbol{x}}(p,q)$ is the *x* component of the vector angular spectrum and is

* Correponding author.
E-mail address: zhouguoquan178@sohu.com (G. Zhou).

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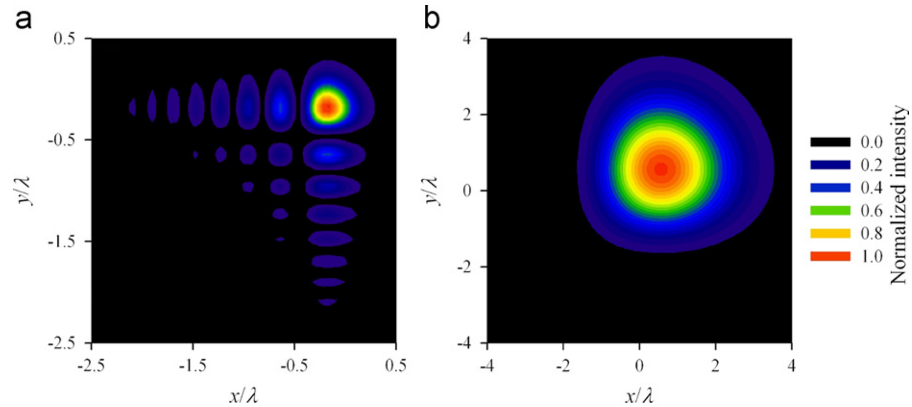

Fig. 1. Contour graph of the normalized intensity distribution of an Airy beam in the source plane: (a) $a=0.1$ and $w_0=0.2\lambda$ and (b) $a=1$ and $w_0=\lambda$.

given by the Fourier transformation of the x component of the initial electric field:

$$A_x(p,q)=\frac{1}{\lambda^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E_x(x,y,0)\exp\left[-ik(px+qy)\right]dxdy$$
$$=\frac{w_0^2}{4\pi\lambda^2}\exp\left[\frac{(a-ikw_0p)^3}{3}+\frac{(a-ikw_0q)^3}{3}\right] \quad (3)$$

The longitudinal electric field in Eq. (2) stems from the divergence theorem of the electric field. By taking the curl of Eq. (2), the corresponding magnetic field of the Airy beam yields

$$\boldsymbol{H}(x,y,z)=\sqrt{\frac{\varepsilon_0}{\mu_0}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}A_x(p,q)\left(-\frac{pq}{\gamma}\boldsymbol{e}_x+\frac{1-q^2}{\gamma}\boldsymbol{e}_y-q\boldsymbol{e}_z\right)$$
$$\times\exp[ik(px+qy+\gamma z)]dpdq \quad (4)$$

where ε_0 and μ_0 are the electric permittivity and the magnetic permeability of vacuum, respectively. $\boldsymbol{e}_y$ is the unit vector in the y-direction. The time dependent factor $\exp(-i\omega t)$ is omitted in Eqs. (2) and (4) where ω is the angular frequency.

As the condition $kr=k(x^2+y^2+z^2)^{1/2}\rightarrow\infty$ is satisfied in the far-field regime, the method of the stationary phase is applicable. Using the method of the stationary phase [24,25], the surface integral of Eq. (2) is shown to have the asymptotic value [26]:

$$\boldsymbol{E}(x,y,z)=\frac{i\lambda}{r}\sum_j\frac{v_j}{(|\alpha_j\beta_j-\delta_j^2|)^{1/2}}\boldsymbol{M}(p_j,q_j)\exp\left[ikrF(p_j,q_j,x,y)\right] \quad \text{as} \quad kr\rightarrow\infty, \quad (5)$$

where $\boldsymbol{M}(p_j,q_j)$ and $F(p_j,q_j,x,y)$ are given by

$$\boldsymbol{M}(p_j,q_j)=\frac{w_0^2}{4\pi\lambda^2}\exp\left[\frac{(a-ikw_0p_j)^3}{3}+\frac{(a-ikw_0q_j)^3}{3}\right]$$
$$\times\left[\boldsymbol{e}_x-\frac{p_j}{(1-p_j^2-q_j^2)^{1/2}}\boldsymbol{e}_z\right], \quad (6)$$

$$F(p_j,q_j,x,y)=\frac{p_jx+q_jy+(1-p_j^2-q_j^2)^{1/2}z}{r}. \quad (7)$$

The stationary points $(p_j,\ q_j)$ are solutions of the following simultaneous equations:

$$\left.\frac{\partial F(p,q,x,y)}{\partial p}\right|_{\substack{p=p_j\\q=q_j}}=0, \quad (8)$$

$$\left.\frac{\partial F(p,q,x,y)}{\partial q}\right|_{\substack{p=p_j\\q=q_j}}=0. \quad (9)$$

Substituting Eq. (7) into Eqs. (8) and (9), we can obtain

$$p_1=x/r,\ q_1=y/r. \quad (10)$$

There is only one stationary point. As a result, the parameters α_1, β_1, δ_1, and v_1 turn out to be

$$\alpha_1=\left.\frac{\partial F^2(p,q,x,y)}{\partial p^2}\right|_{\substack{p=p_1\\q=q_1}}=-1-\frac{x^2}{z^2}, \quad (11)$$

$$\beta_1=\left.\frac{\partial F^2(p,q,x,y)}{\partial q^2}\right|_{\substack{p=p_1\\q=q_1}}=-1-\frac{y^2}{z^2}, \quad (12)$$

$$\delta_1=\left.\frac{\partial F^2(p,q,x,y)}{\partial p\partial q}\right|_{\substack{p=p_1\\q=q_1}}=-\frac{xy}{z^2}, \quad (13)$$

$$v_1=-1. \quad (14)$$

Therefore, the analytical expressions of the propagating electromagnetic field of an Airy beam in the far-field turn out to be

$$\boldsymbol{E}(x,y,z)=-\frac{iw_0^2z}{4\pi\lambda r^2}\exp\left[\frac{(ar-ikw_0x)^3}{3r^3}+\frac{(ar-ikw_0y)^3}{3r^3}+ikr\right]\left(\boldsymbol{e}_x-\frac{x}{z}\boldsymbol{e}_z\right), \quad (15)$$

$$\boldsymbol{H}(x,y,z)=\sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{iw_0^2}{4\pi\lambda r^3}\exp\left[\frac{(ar-ikw_0x)^3}{3r^3}+\frac{(ar-ikw_0y)^3}{3r^3}+ikr\right]$$
$$\times[xy\boldsymbol{e}_x-(x^2+z^2)\boldsymbol{e}_y+yz\boldsymbol{e}_z]. \quad (16)$$

The energy flux distribution of an Airy beam in the far-field is given by

$$<S_z>=\frac{1}{2}\mathrm{Re}[\boldsymbol{E}(x,y,z)\times\boldsymbol{H}^*(x,y,z)]_z$$
$$=\sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{w_0^4z(x^2+z^2)}{32\pi^2\lambda^2r^5}\exp\left(\frac{4a^3}{3}-2ak^2w_0^2\frac{\rho^2}{r^2}\right), \quad (17)$$

where Re denotes taking the real part, and the asterisk means the complex conjugation. The power of an Airy beam in the far-field is found to be

$$P=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}<S_z>dxdy=\sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{w_0^4}{64\pi\lambda^2}$$
$$\times\exp\left(\frac{4a^3}{3}\right)\int_0^1\frac{2-\tau}{\sqrt{1-\tau}}\exp(-2ak^2w_0^2\tau)d\tau. \quad (18)$$

In the above equation, ρ^2/r^2 has been replaced by τ and no approximation has been adopted. To obtain the analytical expression of the power, we first perform the expansion as

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