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Diffraction effect of the injected beam in axisymmetrical structural CO_2 laser

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ABSTRACT

Diffraction effect of the injected beam in axisymmetrical structural CO_2 laser is studied based on the injection-locking principle. The light intensity of the injected beam at the plane where the holophotes lie is derived according to the Huygens–Fresnel diffraction integral equation. And then the main parameters which influence the diffraction light intensity are given. The calculated results indicate that the first-order diffraction signal will play an important role in the phase-locking when the zero-order diffraction cannot reach the folded cavities. The numerical examples are given to confirm the correctness of the results, and the comparisons between the theoretical and the experimental results are illustrated.

1. Introduction

In the laser optics, the diffraction is complex problem. The analytical solution often can not be obtained due to the difficulties in mathematical calculations. Recently, the propagation properties and diffraction effect [1–7] of Gaussian beams through a circular hole (or circular disk) are studied, the research results can afford the references for the propagation transformation of the laser beam.

The phase-locking properties of axisymmetrical structural CO_2 laser [8–10] have been reported. The control beam from the symmetric axis of laser is reflected by the control mirror and injected into the folded cavities. The eigenmodes of the folded cavities will be controlled by injection-mode if the injected signal at the folded cavities is intense enough, and the phase-locking can be fulfilled. In fact, the zero-order diffraction at mirrors M_i could be weak because there is a big angle between the axis of folded cavities M_i-M-M_{i+1} and z-axis (see Fig. 1), therefore, the first-order diffraction effect of the injected beam in the folded cavities should be considered. In order to obtain the controllable diffraction beam, a small hole is placed in the outside of resonator, the radius is a. The beam reflected by control mirror M_6 will diffract at the hole. The diffraction signal can reach folded cavities M_i-M-M_{i+1}, but the diffraction light intensity may be restricted by the control mirror parameters and small hole radius a. Therefore, in order to come true the phase-locking and improve the injected light intensity at folded cavities M_i-M-M_{i+1}. In this paper, we should solve two problems. (1) Confirm the zero-order diffraction can not reach the all reflective mirrors M_i (M_{i+1}) through the numerical calculations, but the first-order diffraction can. (2) How to improve effectively the first-order diffraction light intensity at the folded cavities, and find the main parameters which influence the first-order diffraction. And then the place and light intensity of the first-order diffraction in the folded cavities are studied.

2. Diffraction analysis of the injected beam

In Fig. 1, mirrors M_i and M_0 are the holophotes, M is the output mirror. M_6 is the control mirror, its focus is f, the distance between the control mirror and the output mirror is S. The dashed straight-line denotes the axis of the folded cavities discharge tube. The beam waist radius exported from plane-concave cavity $M-M_0$ [8] is $w_0=1.76$ mm. And then it will be reflected and transformed by the control mirror. Setting the waist radius of the transformed beam is $w_0(f)$, the corresponding complex parameter is $q_0(f)$, the distance between beam waist and control mirror is X. The ray matrix with reference to the output mirror is given by

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & X \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & S \\ 0 & 1 \end{pmatrix}. \tag{1}$$

the beam waist radius $w_0(f)$ and place X according to the ABCD law [11] can be expressed as

$$w_0(f) = \left[\frac{\lambda^2 f^2 w_0{}^2}{\lambda^2 (f-S)^2 + (\pi w_0{}^2)^2} \right]^{1/2}. \tag{2}$$

$$X = \frac{\lambda^2 f S(S-f) + f(\pi w_0^2)^2}{\lambda^2 (f-S)^2 + (\pi w_0^2)^2}. \tag{3}$$

where, the wavelength $\lambda = 10.6$ μm.

The reflected beam will diffract at the small hole, the diffraction light intensity is shown in Fig. 1. FOD (ZOD) denotes the first-order

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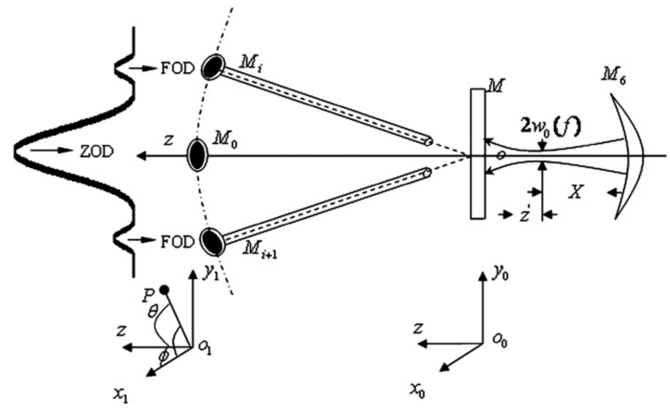

Fig. 1. Axisymmetrical structural CO_2 laser diagram and the diffraction light intensity curve diagram.

(zero-order) diffraction at the all reflective mirror $M_{i,i+1}$ (M_0), respectively. Under the paraxial approximation, setting the distance between beam waist and the hole is $z' \approx S - X$. Therefore, the light field distribution of the incident beam at the hole can be expressed approximately as

$$E(r_0, z') = \frac{w_0(f)}{w(z')} \cdot \exp(-ikz') \cdot \exp\left[-\frac{ik}{2} \cdot \frac{{r_0}^2}{q(z')}\right] \tag{4}$$

where, k is the wave number, the additional phase $\phi(z')$ is omitted.

Setting the distance between the observation plane ($x_1 o_1 y_1$) and the output mirror is z, the field angle of the arbitrary point P to z and x_1 axes are θ and φ, respectively. Therefore, the direction cosine l and m point P to x_1 and y_1 axes can be given by

$$l \approx \sin\theta \cos\phi \tag{5}$$

$$m \approx \sin\theta \sin\phi \tag{6}$$

respectively.

Based on the Huygens–Fresnel diffraction integral equation [12], the light field in plane ($x_1 o_1 y_1$) can be expressed as

$$E(r_1, z) = \frac{i}{\lambda z}\frac{w_0(f)}{w(z')} \cdot \exp\left[-ik(z+z')\right] \cdot \int_0^a \int_0^{2\pi} \exp[ik(lr_0\cos\phi_0 + mr_0 \sin\phi_0)] \times \exp\left[-\frac{ik}{2} \cdot \left(\frac{1}{z} + \frac{1}{q(z')}\right){r_0}^2\right] r_0 \mathrm{d}r_0 \mathrm{d}\phi_0 \tag{7}$$

where, r_1 is the distance between point P and z axis, $w(z')$ is the radius of injected beam at the output mirror and $q(z')$ is the corresponding complex parameter. r_0 and φ_0 are the radial and angular components, respectively.

using the important Bessel function equations

$$J_0(x) = \frac{1}{2\pi}\int_0^{2\pi} \exp(ix\cos\theta)\mathrm{d}\theta. \tag{8}$$

$$\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x). \tag{9}$$

Substituting Eqs. (4)–(6) into Eq. (7), then the field distribution of the observation plane can be given by

$$E(r_1, z) = \frac{ika^2 w_0(f)}{w(z')z} \cdot \exp\left[-ik(z+z')\right] \cdot \exp\left(-\frac{a^2}{\beta}\right) \cdot \sum_{n=1}^{\infty} \left[\frac{J_n(\mu a)}{(\mu a)^n} \cdot \left(\frac{2a^2}{\beta}\right)^{n-1}\right] \tag{10}$$

where, $\mu = kr_1/z$ and β is expresses as

$$\beta = \left\{\frac{ik}{2}\left[\frac{1}{q(z')} + \frac{1}{z}\right]\right\}^{-1} \tag{11}$$

using the property of Bessel function $\lim_{x\to 0} \frac{J_n(x)}{x^n} = \frac{1}{2^n n!}$, the light field in the center ($r_1 = 0$) of observation plane is given by

$$E(0, z) = \frac{ika^2 w_0(f)}{w(z')z} \cdot \exp\left[-ik(z+z')\right] \cdot \exp\left(-\frac{a^2}{\beta}\right) \cdot \sum_{n=1}^{\infty} \left[\frac{1}{2n!} \cdot \left(\frac{a^2}{\beta}\right)^{n-1}\right]. \tag{12}$$

Therefore, the corresponding light intensities can be given by

$$I(r_1, z) = E(r_1, z) \times [E(r_1, z)]^* \tag{13}$$

$$I(0, z) = E(0, z) \times [E(0, z)]^* \tag{14}$$

respectively.

In order to study conveniently, setting $a = \gamma w_0(f)$, γ is the proportional coefficient.

3. Numerical examples and result discussions

The diffraction light fields of the injected beam at the folded cavities are given by Eqs. (10) and (12), $E(r_1, z)$ and $E(0, z)$ will be determined by the parameters (w_0, f, S, z, γ, a). In fact, we know from Eqs. (2) and (3) that $w_0(f)$ and X will influence directly incident field $E(r_0, z')$ and the diffraction field $E(r_1, z)$. Therefore, we should analyze first the two parameters, and the curve diagrams of $w_0(f)$ and X are shown in Figs. 2 and 3, respectively. We know from these figures that the maximum beam waist ($w_0(f) = 0.7668$ mm) is at $S = 40$ cm when $f = 40$ cm. $w_0(f) = 0.7493$ mm and $X = 36.38$ cm when $f = 40$ cm and $S = 20$ cm, the results are coincident with the imaging formula.

What is pointed out especially is that the values of f and S can not be set any value in this paper because the following reasons: (1) The output laser beam from the laser could be considered approximately as the parallel beam in the Rayleigh range ($Z_0 = \pi {w_0}^2/\lambda = 0.918$ m). Therefore, in order to obtain the intense injected signal, $0 < f < Z_0$, in fact, there is a distance between the control mirror and the output mirror [8], and the diffracted hole should be placed the between the two mirrors, therefore, we set $f = 0.4$ m. (2) In order to let the injected beam diffract in the diffracted hole and obtain the intense diffracted signal, the distance S between the control mirror and the output mirror should be slightly larger the focus length f, therefore, we set $S = 0.5$ m.

We know from Eq. (2) that the waist radius of the transformed beam $w_0(f)$ is determined by the parameters (f, S, w_0) when the wavelength $\lambda = 10.6$ μm, and the control mirror is a concave mirror with the convergent function. Therefore, the waist radius of the transformed beam is very small ($w_0(f) \approx 0.7$ mm). In Fig. 2(a), although $w_0(f)$ will increase when focus length f is large, $w_0(f)$ could be considered approximately as an invariant value in the Rayleigh range.

The eigenmodes of the folded cavities will be controlled by injection-mode if the injected signal at the folded cavities is intense, and there is the synchronous phase between the injected-mode and the eigenmodes, therefore, the phase-locking can be come true successfully. We know from the discussions above that the diffraction light intensity will be determined by the parameters (w_0, f, S, z, γ, a). Therefore, in this paper, we will study the influence of these parameters on the relative light intensity $I(r_1, z)/I(0, z)$. The curve diagrams between $I(r_1, z)/I(0, z)$ and r_1 are shown in Fig. 4.

We know from Eqs. (10) and (12) that $E(r_1, z)$ and $E(0, z)$ are determined by the different parameters (w_0, f, S, z, γ, a). Therefore, in comparison with Figs. 4 (a–f), the first-order diffraction will be far away from z axis with the increase of the propagation distance and the diffraction light intensity will become gradually intense. It is suitable for the model having a big angle between the folded