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On the business cycle implications of alternative risk aversion formulations☆

Orhan Torul*

Boğaziçi University, Turkey

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ABSTRACT

In this paper, I investigate the effects of alternative risk aversion formulations on business cycle properties of an otherwise standard real business cycle economy. I first report on the implications of different risk aversion formulations on impulse response functions of real variables, and show that when risk aversion coefficient co-moves counter-cyclically, responses of real variables vary sizeably due to additional wedges both in the intratemporal and the intertemporal margin. Next, I show that formulating the risk aversion coefficient as random walk instead of a deep structural parameter generates better fit with observed volatilities of real variables. Finally, I report that modelling risk aversion coefficient in an endogenously-driven counter-cyclical way improves match with data on real variable correlations.

1. Introduction

Ever since the real business cycle (RBC) revolution, macroeconomics has long formulated key structural fundamentals in the form of *deep parameters*.[1] These fundamentals include the subjective discount rate, Cobb-Douglas production technology, linear capital depreciation rate, functional form of the utility or felicity function, and its associated risk aversion parametrization. A growing body of literature challenges these assumptions and urges to modify the formulation of *deep* fundamentals on different grounds, mainly for the sake of matching empirical patterns better.[2] Particularly, Eeckhoudt et al. (1996), Malmendier and Nagel (2011), Giuliano and Spilimbergo (2014), Bucciol and Zarri (2013), Guiso et al. (2013), Hanaoka et al. (2015) and Mengel et al. (2016), all argue that risk aversion is *not* constant over time, and is either time or state variant with long-lasting persistence. However, so far neither the empirical properties, nor the consequences of alternative formulations of these parameters have been investigated.[3]

In this paper, I address this issue by investigating the business cycle implications of plausible risk aversion formulations in an otherwise standard RBC economy. Specifically, I study the implications of two alternative competing specifications on risk aversion formulation, and compare them with the plain-vanilla RBC economy. Under the first scenario, I formulate that risk aversion features

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* Boğaziçi University, Department of Economics, 34342 Bebek, Istanbul, Turkey.
E-mail address: orhan.torul@boun.edu.tr.

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[1] Among others, see Kydland and Prescott (1982) for more detailed discussion on this issue.

[2] Among others, see Bai et al. (2012) for productive demand shocks, Cho and Cooley (1994) for the incorporation of extensive and intensive labor margins into the utility function to improve on business cycle statistics accuracy, and Fernandez-Villaverde and Rubio-Ramírez (2007) for a broader criticism on modelling "structural" parameters *structurally*, i.e. formulating as state and time-invariant.

[3] The main exception is by Epstein and Zin (1989), which aims to break the link between intertemporal elasticity of substitution and preferences over risk, but does not address the time or state-dependent nature of risk aversion.

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stochasticity over time: while the representative household knows about his *current* risk preferences, he faces uncertainty about his *future* risk aversion, which has an unpredictable exogenous component, along with long-lasting persistence.[4] Accordingly, I model that risk aversion evolves stochastically towards a long-term mean with an autoregressive (of order one) process, and I coin this specification as the "stochastic σ" specification. Under the second competing scenario, following Roemer (1994), Malmendier and Nagel (2011) and Giuliano and Spilimbergo (2014), and Rogerson (1988) who claim that in bad times risk aversion increases and in good times it decreases, I formulate that risk aversion of the representative-agent is negatively related to income (and output).[5] I coin this specification as the "endogenous σ" specification. Throughout my analysis, I employ two parameter sets for each specification. The first parameter set is one where disutility over labor is convex and the Frisch elasticity is set to conventional estimates,[6] and the second parameter set features "indivisible labor" *à la* Hansen (1985) and Rogerson (1988) where representative household is risk-neutral in labor, or equivalently disutility over labor is linear.

I first report on the implications of different risk aversion formulations on the impulse response functions of real variables, and display that endogenizing risk aversion coefficient has notable implications on the responses of real variables to total factor productivity shocks. This finding stems from the fact that endogeneity of risk aversion induces a wedge both in the intratemporal and the intertemporal optimality conditions, which alters how households respond to standard stochasticity. Next, I show that formulating the risk aversion coefficient of consumption as random walk instead of a deep structural parameter generates better fit with observed volatilities of real variables. Finally, I report that modelling risk aversion coefficient in an endogenously-driven counter-cyclical way improves match with data on real variable correlations.

The rest of the paper is organized as follows: in section 2, I describe the model environment, in section 3, I discuss the computational methodology and present the results, in section 4, I conclude.

2. Model environment

2.1. Baseline model

The problem of the benevolent social planner of the RBC economy is to maximize the present discounted life-time utility of the representative household, subject to the economy-wide resource constraint.[7] Formally, the central planner solves:

$$\max_{\{c_t,n_t,k_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \tag{1}$$

subject to

$$c_t + k_t = e^{z_t} f(k_{t-1}, n_t) + (1-\delta)k_{t-1} \tag{2}$$

where c_t denotes consumption, n_t denotes labor (normalized to 1 so that leisure equals $l_t = 1 - n_t$), k_{t-1} denotes capital (as a *state* variable at time t), and z_t denotes total factor productivity, respectively.

Total factor productivity z_t is governed by a stochastic process featuring an error term ε^z_{t+1} and a persistence parameter ρ_z. Specifically, total factor productivity follows:

$$z_{t+1} = (1-\rho_z)\bar{z} + \rho_z z_t + \varepsilon^z_{t+1} \tag{3}$$

where ε^z_{t+1} is distributed normally with zero mean, and a homoskedastic variance σ^2_z, i.e. $\varepsilon^z_t \sim N(0, \sigma^2_z)$.[8] For the remaining parameters, δ refers to the depreciation rate of capital and β refers to the subjective discount factor.

Regarding functional forms, I assume that the utility function features a constant elasticity of (intertemporal) substitution: σ, and a Frisch labor elasticity: $\frac{1}{\upsilon}$, as standard in the RBC literature:

$$u(c,n) = \frac{c^{1-\sigma} - 1}{1-\sigma} - \frac{\psi}{1+\upsilon} n^{1+\upsilon} \tag{4}$$

Further, I assume the production technology follows the standard Cobb-Douglas functional form:

$$f(k,n) = k^{\alpha} n^{1-\alpha} \tag{5}$$

where total output equals $y = e^z f(k,n)$.

The solution to the social planner's problem yields the following *intratemporal* and *intertemporal* margins:

$$\psi n_t^{\upsilon+\alpha} = e^{z_t} c_t^{-\sigma} (1-\alpha) k_{t-1}^{\alpha} \tag{6}$$

$$c_t^{-\sigma} = \beta\, \mathbb{E}_t \left[\left(e^{z_{t+1}} \alpha k_t^{(\alpha-1)} n_{t+1}^{(1-\alpha)} + 1 - \delta \right) c_{t+1}^{-\sigma} \right] \tag{7}$$

where the former margin refers to the consumption-leisure efficiency condition, and the latter refers to the consumption-investment efficiency condition. Also, assuming economy is an autarky, the aggregate resource constraint has to hold:

$$c_t + k_t = (1-\delta)k_{t-1} + e^{z_t} k_{t-1}^{\alpha} n_t^{(1-\alpha)} \tag{8}$$

In order to calculate the deterministic steady-state, one can set the variables to their long-run means, and simplify the system of equations as follows:[9]

$$\psi \bar{n}^{\upsilon+\alpha} = \bar{c}^{-\sigma} (1-\alpha) \bar{k}^{\alpha} \tag{9}$$

$$1 = \beta \left(\alpha \bar{k}^{(\alpha-1)} \bar{n}^{(1-\alpha)} + 1 - \delta \right) \tag{10}$$

4 Hanaoka et al. (2015) show that i) 2011 earthquake significantly affect risk preferences of Japanese men, and ii) even five years after the earthquake, their modified risk preferences persist.

5 In brief, the foundation of this argument is based on the grounds that during times of substantial negative shocks, as in the case of the second world war or the great depression, households tend to get more risk-averse and favor social insurance more.

6 Note that Chetty et al. (2011) propose the use of a Frisch elasticity of 0.75 for macroeconomic models, and I set the Frisch elasticity in the benchmark parameter set accordingly.

7 The model features no government and externalities. Accordingly, the solution to the social planner's problem is equivalent to the competitive equilibrium by the first welfare theorem. Further, the prices are implicitly defined as $w_t = e^{z_t} f_n(k_{t-1}, n_t)$ and $r_t = e^{z_t} f_k(k_{t-1}, n_t) - \delta$, where w_t denotes real wage, r_t denotes real return of physical capital, and $f_n(\cdot)$ and $f_k(\cdot)$ denotes partial derivative of the production function $f(\cdot)$ with respect to labor and physical capital, respectively.

8 Accordingly, the distributional properties of ε^z_t implies that at the steady-state $e^{\bar{z}} = 1$ holds true.

9 After calculating the deterministic steady-state, I derive the decision rules and resultant business cycle statistics around the deterministic steady-state via Schmitt-Grohé and Uribe (2004) second-order local approximation algorithm.

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