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Relativistic self-focusing of cosh-Gaussian laser beams in a plasma

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ABSTRACT

In the present paper, we have proposed to exploit an analysis of a self-consistent, steady state, theoretical model, which explains the propagation of cosh-Gaussian laser beams in a plasma. The nonlinearity we have considered is of relativistic type. Using the expression for the dielectric function, we have setup the differential equation for beam-width parameter using the WKB and paraxial approximations. The effect of decentred parameter of the beam on the critical curve and the dependence of the beam-width on the distance of propagation have been specifically considered. The results have been presented in the form of graphs and discussed.

1. Introduction

The phenomenon of self-focusing [1,2], associated with the interaction of intense laser beams with plasmas, has been a subject of considerable interest for physicists over a few decades. Since the nonlinear effects in plasmas are highly sensitive to the irradiance distribution along the wavefront of the beam, which is significantly affected by self-focusing, the magnitude of all nonlinear processes in plasmas is affected by this phenomenon. Thus, self-focusing is a topic of immense importance in emerging areas such as optical harmonic generation [3–5], inertial confinement fusion [6–8], ionospheric modification [9–11] and laser-electron acceleration [12–14]. So far the main thrust of experimental and theoretical investigations on self-focusing of laser beams in plasmas has been directed to the study of nonlinear propagation characteristics of a Gaussian beam [2,15–35]. Nevertheless, a few studies have been made on self-focusing of super Gaussian beams [36], degenerate modes of beams [37], elliptic Gaussian beams [38,39], Bessel beams [40], Hermite–Gaussian beams [41], Laguerre–Gaussian beams [42] and dark hollow Gaussian beams [43–45].

Apart from these, great interest has recently been evinced in production and propagation of decentred Gaussian beams, usually known as cosh-Gaussian beams on account of their wide and attractive applications in complex optical systems [46–50] and turbulent atmosphere [51–53]. The propagation properties of cosh-Gaussian laser beams are important technological issues, since these beams possesses high power in comparison to that of a Gaussian beam [54]. In relatively recent studies, propagation of cosh-Gaussian laser beams in various nonlinear media has been investigated in detail [55–60]. In the present investigation, authors have studied evolution of cosh-Gaussian laser beams in relativistic plasma through parabolic equation approach. For the sake of simplicity, in common with most investigations the present analysis makes use of the paraxial approximation. In Section 2, we present the field distribution of cosh-Gaussian beams, nonlinear dielectric constant of the plasma under relativistic nonlinearity and derive the differential equation governing the nature of self-focusing of cosh-Gaussian beams in plasma. A discussion of results is given in Section 3. Finally a brief conclusion is added in Section 4.

2. Propagation of cosh-Gaussian laser beam

Consider the propagation of cosh-Gaussian laser beam of angular frequency ω in homogeneous plasma along the z direction. The initial electric field distribution of the beams are given by

$$E(r,0)=E_0\cosh(\Omega_0 r)\exp\left(-\frac{r^2}{w_0^2}\right), \quad (1)$$

where r is the radial coordinate of the cylindrical coordinate system, w_0 is the waist width of the Gaussian amplitude distribution, E_0 is the amplitude at the central position of $r=z=0$ and Ω_0 is the parameter associated with the hyperbolic cosine function, also called the cosh factor.

On the other hand, we can express Eq. (1) as follows

$$E(r,0)=\frac{E_0}{2}\exp\left(\frac{b^2}{4}\right)\left\{\begin{array}{c}\exp[-((r/w_0)+(b/2))^2]\\+\exp[-((r/w_0)-(b/2))^2]\end{array}\right\}, \quad (2)$$

where $b=w_0\Omega_0$ is the decentred parameter, also called the normalised modal parameter.

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The electric vector E of the beam satisfies the wave equation

$$\nabla^2 E - \nabla(\nabla E) + \frac{\omega^2 \varepsilon E}{c^2} = 0, \tag{3}$$

with

$$\varepsilon = \varepsilon_0 + \Phi(EE^*), \tag{4}$$

where ε_0 and Φ are the linear and nonlinear parts of the dielectric constant, respectively, and

$$\varepsilon_0 = 1 - \frac{\omega_{pe}^2}{\omega^2}. \tag{5}$$

here ω_{pe} is the plasma frequency, given by $\omega_{pe}^2 = 4\pi n_e e^2/m_0$ where e and m_0 are the charge and rest mass of the electron, respectively, and n_e is the density of plasma electrons in the absence of laser beam. The relativistic general expression for the plasma frequency is given by

$$\omega_{pe}^2 = \frac{\omega_p^2}{\gamma}, \tag{6}$$

where γ is the relativistic factor given by

$$\gamma = (1 + \alpha EE^*)^{-1/2}, \tag{7}$$

where $\alpha = e^2/m_0^2 \omega^2 c^2$. We can express the nonlinear part of the dielectric constant by the following equation

$$\Phi(EE^*) = \frac{\omega_p^2}{\omega^2}[1 - (1 + \alpha EE^*)^{-1/2}]. \tag{8}$$

In Wentzel–Kramers–Brillouin (WKB) approximation, the second term of Eq. (3) can be neglected and the electric vector E of the beam satisfy the following equation

$$\nabla^2 E + \frac{\omega^2}{c^2}\varepsilon E = 0. \tag{9}$$

It should be mentioned that neglecting the term $\nabla(\nabla E)$ in Eq. (3) is justified when $(c^2/\omega^2)|(1/\varepsilon)\nabla^2 \ln\varepsilon| \ll 1$. Now, $E = A(r,z)\exp(-ikz)$ is introduced, where $A(r,z)$ is the complex function of its argument. The behaviour of the complex amplitude $A(r,z)$ is described by the parabolic equation obtained from the wave Eq. (9) in the WKB approximation assuming that the variations in the z direction are slower than those in radial direction.

$$-2ik\frac{\partial A}{\partial z} + \nabla_\perp^2 A + \frac{\omega^2}{c^2}\Phi(EE^*)A = 0. \tag{10}$$

To solve Eq. (10), we express A as

$$A = A_0(r,z)\exp(-ikS), \tag{11}$$

where A_0 and S are real functions of r and z (S being the eikonal of the beam). Substituting Eq. (11) for A in Eq. (10) and separating the real and imaginary parts, one can obtain

$$2\left(\frac{\partial S}{\partial z}\right) + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{k^2 A_0}\nabla_\perp^2 A_0 + \frac{\omega_p^2}{\omega^2 \varepsilon_0}[1 - (1 + \alpha EE^*)^{-1/2}], \tag{12}$$

and

$$\frac{\partial A_0^2}{\partial z} + \left(\frac{\partial S}{\partial r}\right)\frac{\partial A_0^2}{\partial r} + A_0^2\left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) = 0. \tag{13}$$

Following Akhmanov et al. [1] and Sodha et al. [2], the solution for E can be written as

$$E = A_0(r,z)\exp[-ik(S+z)],$$

$$A_0^2 = \frac{E_0^2}{4f^2}\exp\left(\frac{b^2}{2}\right)\left\{\begin{array}{l}\exp\left[-2\left(\frac{r}{fw_0} + \frac{b}{2}\right)^2\right] + \exp\left[-2\left(\frac{r}{fw_0} - \frac{b}{2}\right)^2\right] \\ +2\exp\left[-\left(\frac{2r^2}{f^2 w_0^2} + \frac{b^2}{2}\right)\right]\end{array}\right\},$$

$$S = \frac{r^2}{2}\beta(z) + \phi(z), \tag{14}$$

where $\beta(z) = (1/f)(df/dz)$ and $k = \omega\varepsilon_0^{1/2}/c$. The parameter β^{-1} may be interpreted as the radius of the curvature of the beam. f is the dimensionless beam-width parameter described by the differential equation

$$\frac{d^2 f}{d\eta^2} = \frac{12 - 12b^2 - b^4}{3f^3} - \left(\frac{w_0\omega}{c}\right)^2\left(\frac{\omega_p}{\omega}\right)^2\frac{\alpha E_0^2}{f^3}\exp\left(\frac{b^2}{2}\right)\left[1 + \frac{\alpha E_0^2}{f^2}\right]^{-3/2}, \tag{15}$$

where $\eta = z/kw_0^2$ is the dimensionless distance of propagation. Eq. (15) can be solved numerically with appropriate boundary conditions. One can take $f=1$ and $df/dz=0$, corresponding to an initial plane wavefront.

3. Results and discussion

Eq. (15) is a nonlinear ordinary differential equation governing the behaviour of dimensionless beam-width parameter f as a function of distance of propagation η. The first term on the right-hand-side of this equation has its origin in the Laplacian $\nabla_\perp^2$ appearing in the evolution Eq. (10). When high intense power laser beam is used, the second term in Eq. (15) arises due to the relativistic nonlinear effect resulting from the relativistic mass correction and depends on decentred parameter b, intensity factor αE_0^2, relative plasma density ω_p/ω, etc. The diffraction term leads to the diffractional divergence of the beam, while nonlinear term is responsible for self-focusing of the beam due to the relativistic effect. It is interesting to mention here that both diffractional divergence and self-focusing depends critically on the decentred parameter b of the beam. Thus the fate the laser beam is ultimately determined by the magnitude of b values. If the first term on right-hand-side of Eq. (15) dominates over the second term, the beam diverges, while opposite is true when the second term exceeds the first one. We have solved Eq. (15) numerically with following set of parameters: $\omega = 1.778 \times 10^{20}$ rad/s, $n_e = 4 \times 10^{19}$ cm^{-3}, $\alpha E_0^2 = 0.1$–0.3, $w_0 = 20\,\mu m$, $\omega_p/\omega = 1 \times 10^{-6}$–$3 \times 10^{-6}$, $b = 0$–3. The results are depicted in the form of graphs.

Fig. 1 present the variation of beam-width parameter f with the dimensionless distance of propagation η for four values of decentred parameter $b = 0$, 1, 2 and 3. It is observed form Fig. 1 that oscillatory self-focusing takes place for $b = 0$ and 1 while for $b > 1$ beam-width parameter f decreases steeply. It is also important to notice that self-focusing occurs at shorter propagation distance for higher b values and focusing is more pronounced for higher b values and thus supports the results of Gill et al. [59] with weakly relativistic ponderomotive regime through variational approach. This is due to the fact that as b value increases,

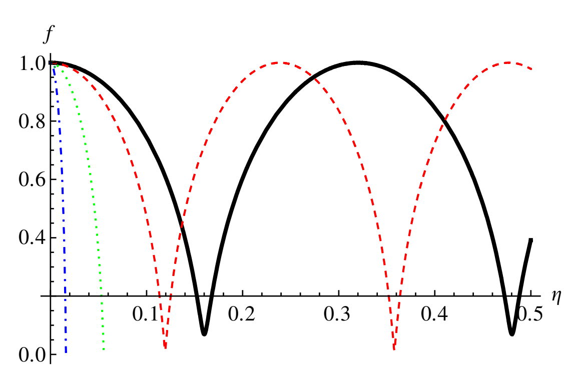

Fig. 1. Variation of beam-width parameter f with dimensionless distance of propagation η for various values of decentred parameter b. The parameters are: $\omega = 1.778 \times 10^{20}$ rad/s, $w_0 = 20\,\mu m$, $n_e = 4 \times 10^{19}$ cm^{-3}, $\omega_p/\omega = 2 \times 10^{-6}$, $\alpha E_0^2 = 0.1$. Solid curve ($b=0$), dashed curve ($b=1$), doted curve ($b=2$), dot dashed curve ($b=3$).