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Propagation of a vectorial Laguerre–Gaussian beam beyond the paraxial approximation

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Abstract

Based on the vectorial Rayleigh–Sommerfeld integrals, the analytical propagation expression of a vectorial Laguerre–Gaussian beam beyond paraxial approximation is presented. The far field expression and the scalar paraxial result are obtained as special cases of the general formulae. According to the analytical representation, the light intensity distribution of the vectorial Laguerre–Gaussian beam is depicted in the reference plane. The light intensity distribution of a vectorial Laguerre–Gaussian beam with $\cos m\varphi$ is also compared with that of a vectorial Laguerre–Gaussian beam with $\sin m\varphi$.

Keywords: Vectorial Rayleigh–Sommerfeld integrals; Beam propagation; Beyond the paraxial approximation

1. Introduction

As is well known, the cylindrically symmetric higher order modes of laser cavities with spherical mirrors are Laguerre–Gaussian beams [1]. Therefore, the Laguerre–Gaussian beam receives considerable interest [2–5]. Usually, the description of a Laguerre–Gaussian beam is by the approximate solution of Helmholtz equation. The propagation properties of Laguerre–Gaussian beams have been studied extensively within the framework of the paraxial approximation. Within the paraxial approximation, the longitudinal component of Laguerre–Gaussian beam vanishes, and there is no restriction within the two transverse components. As a result, it does not satisfy Maxwell's equations and loses its inherent vectorial property, which results in an apparent paradox theoretically. Moreover, the paraxial approximation is invalid for beams with a large divergent angle or a small spot size that is comparable with the light wavelength. Therefore, the interest in extending the understanding of the Laguerre–Gaussian beam to the vectorial nonparaxial regime stems from both theoretical and practical aspects. Various approaches have been proposed to study the beam propagation beyond the paraxial approximation [6–9], one of which, namely the vectorial Rayleigh–Sommerfeld integrals method, is a suitable one [10,11]. Accordingly, the description of the vectorial Laguerre–Gaussian beam is directly derived from the vectorial Rayleigh–Sommerfeld integrals in the present paper. The analytical formulas for the nonparaxial propagation of vectorial Laguerre–Bessel–Gaussian beams have been derived [12]. The influences of two parameters f and α on the propagation behavior have also been analyzed. Though the obtained results can be reduced to those of the cases for vectorial Lauguerre–Gaussian and Bessel–Gaussian beams, the propagation expression for the vectorial Laguerre–Gaussian beam is too complicated. The purpose of the present paper is to present relatively concise expression. Moreover, the influence of the angle-dependent relation on the propagation behavior is also investigated.

2. Propagation of vectorial Laguerre–Gaussian beam beyond paraxial approximation

As TE polarization is a familiar case, the vectorial Laguerre–Gaussian beam is treated to be linearly polarized in the x-direction. The z-axis is taken to be the propagation axis. The vectorial Laguerre–Gaussian beam at the source

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plane $z = 0$ is described by

$$\begin{pmatrix} E_x(\rho_0, 0) \\ E_y(\rho_0, 0) \end{pmatrix} = \begin{pmatrix} \left(\frac{\sqrt{2}\rho_0}{w_0}\right)^m L_n^m\left(\frac{2\rho_0^2}{w_0^2}\right) \exp\left(-\frac{\rho_0^2}{w_0^2}\right) \cos m\varphi_0 \\ 0 \end{pmatrix}, \tag{1}$$

where w_0 is the Gaussian beam waist, and L_n^m is the associated Laguerre polynomial. n and m are the radial and angular mode numbers. (ρ_0,φ_0) denotes the transverse coordinates in the cylindrical coordinate system. $\rho_0 = (x_0^2 + y_0^2)^{1/2}$, and $\varphi_0 = \tan^{-1}(y_0/x_0)$. The time-dependent factor $\exp(-i\omega t)$ is omitted in Eq. (1), and ω is the circular frequency. By using the vectorial Rayleigh–Sommerfeld integrals, the vectorial Laguerre–Gaussian beam propagating toward half free space $z>0$ turns out to be [10]

$$E_x(\boldsymbol{r}) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_x(\rho_0, 0)\frac{\partial \boldsymbol{G}(\boldsymbol{r}, \boldsymbol{\rho}_0)}{\partial z}\,dx_0\,dy_0, \tag{2a}$$

$$E_y(\boldsymbol{r}) = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_y(\rho_0, 0)\frac{\partial \boldsymbol{G}(\boldsymbol{r}, \boldsymbol{\rho}_0)}{\partial z}\,dx_0\,dy_0, \tag{2b}$$

$$E_z(\boldsymbol{r}) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(E_x(\rho_0, 0)\frac{\partial \boldsymbol{G}(\boldsymbol{r}, \boldsymbol{\rho}_0)}{\partial x} + E_y(\rho_0, 0)\frac{\partial \boldsymbol{G}(\boldsymbol{r}, \boldsymbol{\rho}_0)}{\partial y}\right)dx_0\,dy_0, \tag{2c}$$

where

$$\boldsymbol{G}(\boldsymbol{r}, \boldsymbol{\rho}_0) = \frac{\exp(ik|\boldsymbol{r} - \boldsymbol{\rho}_0|)}{|\boldsymbol{r} - \boldsymbol{\rho}_0|}, \tag{3}$$

$\boldsymbol{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, and $\boldsymbol{\rho}_0 = x_0\mathbf{i} + y_0\mathbf{j}$, where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are three unit vectors in the Cartesian coordinate system. $k = 2\pi/\lambda$ is the wave number. λ is the incident wavelength. The following approximation is valid beyond the paraxial approximation [13–15]

$$|\boldsymbol{r} - \boldsymbol{\rho}_0| = r + \frac{\rho_0^2 - 2xx_0 - 2yy_0}{2r}, \tag{4}$$

where $r = (x^2 + y^2 + z^2)^{1/2} = (\rho^2 + z^2)^{1/2}$. Replacing $|\boldsymbol{r}-\boldsymbol{\rho}_0|$ of the exponential part in Eq. (2) by Eq. (4) and other terms by r, one can obtain the propagating vectorial Laguerre–Gaussian beam as follows:

$$E_x(\boldsymbol{r}) = -\frac{iz}{\lambda r^2}\exp(ikr)\int_0^{\infty}\int_0^{2\pi} E_x(\rho_0, 0)\exp\left(\frac{ik\rho_0^2}{2r}\right) \times \exp\left(-\frac{ik\rho\rho_0}{r}\cos(\varphi_0 - \varphi)\right)\rho_0\,d\rho_0\,d\varphi_0, \tag{5}$$

$$E_y(\boldsymbol{r}) = 0, \tag{6}$$

$$E_z(\boldsymbol{r}) = \frac{i}{\lambda r^2}\exp(ikr)\int_0^{\infty}\int_0^{2\pi}(\rho\cos\varphi - \rho_0\cos\varphi_0)E_x(\rho_0, 0) \times \exp\left(\frac{ik\rho_0^2}{2r}\right)\exp\left(-\frac{ik\rho\rho_0}{r}\cos(\varphi_0 - \varphi)\right)\rho_0\,d\rho_0\,d\varphi_0, \tag{7}$$

where $\varphi = \tan^{-1}(y/x)$. Under the integration process, the following integral formula is satisfied

$$J_v(x) = \frac{(-i)^v}{2\pi}\int_0^{2\pi}\exp(ix\cos\theta + iv\theta)\,d\theta, \tag{8}$$

where J_v is the vth order Bessel function of the first kind, and v is an arbitrary integer. Therefore, the x and z components of the vectorial Laguerre–Gaussian beam can be expressed in integral form

$$E_x(\boldsymbol{r}) = -\frac{i^{-m+1}kz}{r^2}\exp(ikr)T_1\cos m\varphi, \tag{9}$$

$$E_z(\boldsymbol{r}) = \frac{i^{-m+1}k}{r^2}\exp(ikr)\left\{T_1 x\cos m\varphi + \frac{i[T_2\cos(m-1)\varphi - T_3\cos(m+1)\varphi)]}{2}\right\}, \tag{10}$$

where T_1, T_2 and T_3 given by

$$T_1 = \int_0^{\infty}\left(\frac{\sqrt{2}\rho_0}{w_0}\right)^m L_n^m\left(\frac{2\rho_0^2}{w_0^2}\right)\exp(-\beta\rho_0^2)J_m\left(\frac{k\rho\rho_0}{r}\right)\rho_0\,d\rho_0, \tag{11}$$

$$T_2 = \int_0^{\infty}\left(\frac{\sqrt{2}\rho_0}{w_0}\right)^m L_n^m\left(\frac{2\rho_0^2}{w_0^2}\right)\exp(-\beta\rho_0^2)J_{m-1}\left(\frac{k\rho\rho_0}{r}\right)\rho_0^2\,d\rho_0, \tag{12}$$

$$T_3 = \int_0^{\infty}\left(\frac{\sqrt{2}\rho_0}{w_0}\right)^m L_n^m\left(\frac{2\rho_0^2}{w_0^2}\right)\exp(-\beta\rho_0^2)J_{m+1}\left(\frac{k\rho\rho_0}{r}\right)\rho_0^2\,d\rho_0, \tag{13}$$

with $\beta = l/w_0^2$, $l = 1 - iz_r/r$, and $z_r = kw_0^2/2$ is the confocal parameter. By applying the mathematical integral formulae

$$\int_0^{\infty} x^{m+1}\exp(-\beta x^2)L_n^m(\alpha x^2)J_m(yx)\,dx = 2^{-m-1}\beta^{-m-n-1}(\beta - \alpha)^n y^m\exp\left(-\frac{y^2}{4\beta}\right)L_n^m\left(\frac{\alpha y^2}{4\beta(\alpha - \beta)}\right), \tag{14}$$

Eq. (11) can be expressed in the analytical form as

$$T_1 = \frac{(-1)^n}{k}\frac{rw_0}{w(r)}\left(\frac{\sqrt{2}\rho}{w(r)}\right)^m\exp\left(-\frac{\rho^2}{w^2(r)} + i\Psi\right)L_n^m\left(\frac{2\rho^2}{w^2(r)}\right). \tag{15}$$

where $\Psi = (z_r\rho^2/rw^2(r)) + (2n + m + 1)\tan^{-1}(z_r/r)$, and $w(r) = w_0(1 + r^2/z_r^2)^{1/2}$ is the beam waist. Therefore, the analytical expression of x component for vectorial Laguerre–Gaussian beam beyond paraxial approximation reads as

$$E_x(\boldsymbol{r}) = (-1)^{n+1}i^{-m+1}\frac{z}{r}\frac{w_0}{w(r)}\left(\frac{\sqrt{2}\rho}{w(r)}\right)^m \times \exp\left(-\frac{\rho^2}{w^2(r)} + ikr + i\Psi\right)L_n^m\left(\frac{2\rho^2}{w^2(r)}\right)\cos m\varphi. \tag{16}$$