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# Combined finite element analysis and subproblem approximation method for the design of ultrasonic motors

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ABSTRACT

This paper presents a design method of ultrasonic motors using finite element analysis and subproblem approximation method. It adopts the finite element method for modal analysis and harmonic analysis of the motor, and subproblem approximation method for searching the optimal solution of design variables. To verify the effectiveness of the design method, the proposed design method is applied to the design of a rod shape ultrasonic motor. The motor structure is determined using the proposed method. A prototype ultrasonic motor is manufactured using the design results and measured experimentally. The results demonstrate that the design method is effective and available for the design of ultrasonic motors.


## 1. Introduction

Ultrasonic motors offer many unique merits such as high torque at a low speed with a high efficiency, frictional locking at the power-off stage, quick response to driving electric signal, precise positioning and no magnetic interference. Because of these features, many different types of ultrasonic motors have been developed in various application fields [1–4].

Many ultrasonic motors use two or three stator vibration modes to generate the elliptical motion of the stator's surface particles, which drives the rotor or slide. To successfully design an ultrasonic motor, it is necessary to consider the following factors: working mode frequency, the amplitude of the vibrations, motor structures, geometrical dimensions, material parameters, the electromechanical coupling of piezoelectric ceramics. The trial and error method is commonly used for solving the design problem, but it is time consuming and inefficient.

The design of ultrasonic motors has been investigated in recent studies. The genetic algorithm method was used to design ultrasonic motors in [5]. An optimization design method using constrained variable metric algorithm was proposed for a three-degree of freedom ultrasonic motor stator in [6]. Although these optimization methods work well, their implementation and programming are complicated and time consuming. Fernandez et al. introduced a standing wave ultrasonic linear motor and a factorial design method to optimize deformation amplitude of the stator [7]. In [8], a design method using particle swarm optimization and finite element method was developed for ultrasonic motors. They aim for simple problems such as increasing the deformation amplitude or optimizing the frequency difference. However, in many engineering design tasks, multiobjective design must be met. Particle swarm optimization is computationally inexpensive and easier to implement than genetic algorithms. But one common problem of particle swarm optimization and genetic algorithms is premature convergence [9–12].

In this paper, a novel design method, using finite element analysis and subproblem approximation method, is proposed and applied to the design of an ultrasonic motor. Finite element method is applied to modal analysis and harmonic analysis of the motor. Subproblem approximation method is used to locate optimum design of design variables.

Subproblem approximation method replaces the dependant variables through the least squares fitting approximation process. A constrained problem is formulated into a basic unconstrained problem by using penalty functions. The penalized functions are then minimized until the convergence is reached or the iterations are terminated. Subproblem approximation method possesses the ability to find the global optimum in full design space. Moreover, it offers distinct advantages in many engineering problems, combining simplicity, satisfactory accuracy and efficiency of computation [13–15].

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## 2. Subproblem approximation method

A problem of optimum design can be stated as follows

$$\text{Min.} \quad F = F(\boldsymbol{x}) \tag{1}$$

Subject to

$$\begin{aligned} & g_i(\boldsymbol{x}) \le \bar{g}_i \quad (i = 1, 2, 3, \ldots, m_1) \\ & \underline{h}_i \le h_i(\boldsymbol{x}) \quad (i = 1, 2, 3, \ldots, m_2) \\ & \underline{w}_i \le w_i(\boldsymbol{x}) \le \bar{w}_i \quad (i = 1, 2, 3, \ldots, m_3) \end{aligned} \tag{2}$$

where $g_i$, $h_i$, $w_i$ are the state variables containing the design, $m_1$, $m_2$, $m_3$ the number of state variables.

The vector of design variables is indicated as

$$\boldsymbol{x} = x_1, x_2, x_3, \ldots, x_n \tag{3}$$

Subproblem approximation method is a good way to solve the constrained optimization problem. In the optimization algorithms, the constrained optimization problem is converted to an unconstrained problem using penalty functions [16–18].

The dependent variables can be expressed by approximations as

$$\begin{aligned} & \hat{F}(\boldsymbol{x}) = F(\boldsymbol{x}) + error \\ & \hat{g}(\boldsymbol{x}) = g(\boldsymbol{x}) + error \\ & \hat{h}(\boldsymbol{x}) = h(\boldsymbol{x}) + error \\ & \hat{w}(\boldsymbol{x}) = w(\boldsymbol{x}) + error \end{aligned} \tag{4}$$

The most complex form that the approximations can take on is a fully quadratic representation with cross terms.

$$\hat{F} = a_0 + \sum_{i}^{n} a_i x_i + \sum_{i}^{n} \sum_{j}^{n} b_{ij} x_i x_j \tag{5}$$

where $a_i$ and $b_{ij}$ are coefficients whose value is determined by the weighted least squares technique.

With function approximations available, the constrained minimization problem is recast as follows

$$\text{Min.} \quad \hat{F} = \hat{F}(\boldsymbol{x}) \tag{6}$$

Subject to

$$\begin{aligned} & \underline{x}_i \le x \le \bar{x}_i \quad (i = 1, 2, 3, \ldots, n) \\ & \hat{g}_i(\boldsymbol{x}) \le \bar{g}_i + \alpha_i \quad (i = 1, 2, 3, \ldots, m_1) \\ & \underline{h}_i - \beta_i \le \hat{h}_i(\boldsymbol{x}) \quad (i = 1, 2, 3, \ldots, m_2) \\ & \underline{w}_i - \gamma_i \le \hat{w}_i(\boldsymbol{x}) \le \bar{w}_i + \gamma_i \quad (i = 1, 2, 3, \ldots, m_3) \end{aligned} \tag{7}$$

where $\alpha_i$, $\beta_i$, $\gamma_i$ represent the state variables related parameters after function approximation available.

The constrained minimization problem in Eq. (6) is converted to the unconstrained problem using penalty functions leading to the following subproblem statement

$$\text{Min.} \quad F(\boldsymbol{x}, P_k) = \hat{F} + F_o P_k \left( \sum_{i=1}^{n} X(x_i) + \sum_{i=1}^{m_1} G(\hat{g}_i) + \sum_{i=1}^{m_2} H(\hat{h}_i) + \sum_{i=1}^{m_3} W(\hat{w}_i) \right) \tag{8}$$

where $X$ is the penalty function used to enforce design variable constraints, $G$, $H$ and $W$ are penalty functions for state variable constraints. A sequential unconstrained minimization technique is used to solve Eq. (8) at each design iteration.

Subproblem approximation iterations continue until convergence is achieved. Convergence is assumed when either the present design set $\boldsymbol{x}^{(j)}$, or the previous design set $\boldsymbol{x}^{(j-1)}$, or the best design set $\boldsymbol{x}^{(b)}$, is feasible and one of the following conditions is satisfied.

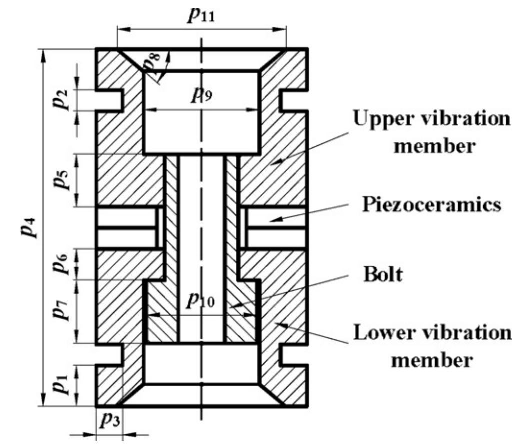


**Fig. 1.** Stator structure and its parameters of the rod shape motor.

$$\left| F^{(j)} - F^{(j-1)} \right| \le \tau \tag{9}$$

$$\left| F^{(j)} - F^{(b)} \right| \le \tau \tag{10}$$

$$\left| x_i^{(j)} - x_i^{(j-1)} \right| \le \rho_i \quad (i = 1, 2, 3, \ldots, n) \tag{11}$$

$$\left| x_i^{(j)} - x_i^{(b)} \right| \le \rho_i \quad (i = 1, 2, 3, \ldots, n) \tag{12}$$

where $\tau$ and $\rho_i$ are respectively objective function and design variable tolerances. If the satisfaction of Eqs. (9)–(12) is realized, then termination occurs.

## 3. Application of finite element analysis and subproblem approximation method for an ultrasonic motor

### 3.1. Description of design problem

The proposed design method using finite element analysis and subproblem approximation method is applied to the design of a traveling wave type rod shape ultrasonic motor.

The motor is composed of one stator and two rotors. The operating vibration modes of the designed motor are two orthogonal first bending vibration modes. In the stator, eight piezoelectric ceramic disks are sandwiched between two vibration members with a tightening bolt. When electrical energy is fed to the piezoceramics, the bending modes of the stator will be excited by the electromechanical energy conversion elements. Then the particles in the end surface of the stator will make elliptical movements, thereby rotors pressed on the stator are driven by friction. The parametrization of the stator is shown in Fig. 1.

A sensitivity analysis is carried out to analyze the influence of each parameter on the operating vibration mode frequency and the amplitude of the stator's surface particles. The most influential design parameters are selected as design variables. Furthermore, the dimension constraint of other motor components is considered. Finally, design parameters $p_1$, $p_2$, $p_3$, $p_4$ are selected as design variables [19].

In order to increase the output characteristics and efficiency of the ultrasonic motor, motor design should satisfy the following design requirements [6–7,19–20]:

1. High rotational speed coefficient. According to the driving principle of the motor, its rotational speed is directly proportional to the amplitude of the driving points and the exciting frequency.