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Extreme daily increases in peak electricity demand: Tail-quantile estimation

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HIGHLIGHTS

- ► Policy makers should design demand response strategies to save electricity.
- ► Peak electricity demand is influenced by tails of probability distributions.
- ► Both the GSP and the GPD are a good fit to the data.
- ► Accurate assessment of level and frequency of extreme load forecasts is important.

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ABSTRACT

A Generalized Pareto Distribution (GPD) is used to model extreme daily increases in peak electricity demand. The model is fitted to years 2000–2011 recorded data for South Africa to make a comparative analysis with the Generalized Pareto-type (GP-type) distribution. Peak electricity demand is influenced by the tails of probability distributions as well as by means or averages. At times there is a need to depart from the average thinking and exploit information provided by the extremes (tails). Empirical results show that both the GP-type and the GPD are a good fit to the data. One of the main advantages of the GP-type is the estimation of only one parameter. Modelling of extreme daily increases in peak electricity demand helps in quantifying the amount of electricity which can be shifted from the grid to off peak periods. One of the policy implications derived from this study is the need for day-time use of electricity billing system similar to the one used in the cellular telephone/and fixed line-billing technology. This will result in the shifting of electricity demand on the grid to off peak time slots as users try to avoid high peak hour charges.

1. Introduction

Modelling of extreme daily increases in peak electricity demand is very important for load forecasters in the electricity sector. Peak electricity demand is an energy policy concern for all economies throughout the world, causing blackouts and increasing the cost of electricity for consumers (Strengers, 2012). This has resulted in many economies in the designing of energy efficient and demand side management strategies to either redistribute or reduce energy demand during peak periods. We define daily increase in peak electricity demand as the positive day-to-day change in daily peak demand (DPD), where DPD is the maximum hourly demand in a 24-hour period. Extreme daily increase in peak electricity demand is therefore positive day-to-day change above a sufficiently high threshold. The demand for electricity forms the basis for power system planning, power security and supply reliability (Ismail et al., 2009). This involves finding the optimal day-to-day operation of a power plant. It is therefore important to have an accurate assessment of the level and frequency of future extreme day-to-day increases in peak electricity demand. Peak electricity demand is subject to a range of uncertainties, including population growth, changing technology, economic conditions, prevailing weather conditions as well as the general randomness in individual usage (Hyndman and Fan, 2010).

Extreme value theory (EVT) is used in this paper to investigate whether extreme daily increases such as the one experienced in May 2007 in South Africa is truly an extreme low-probability event or is one which will appear on a regular basis. The distribution of extreme daily increases in peak electricity is modelled using the Generalized Pareto Distribution (GPD). A comparative analysis is then done using the Generalized Single Pareto (GSP) distribution which has one parameter to estimate (Verster and De Waal, 2011). Modelling of extreme peak electricity demand is important for load forecasters and system planners for planning and scheduling of likely maximum daily increases of peak

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electricity demand. Fitting a GPD to exceedances over a sufficiently large threshold is discussed in literature. Castillo and Hadi (1997) discussed the fitting of the GPD to a given set of data. The estimation of the two parameters of the GPD which are the shape and scale parameters is usually not easy. In their paper, Castillo and Hadi (1997) proposed a method for estimating the parameters and quantiles of the GPD. Their proposed method worked well over a wide range of parameter values. Hosking and Wallis (1987) used GPD in modelling annual maximum floods. Recent work includes that of Verster and De Waal (2011) who showed that the tail of a Generalized Burr-Gamma (GBG) distribution can be approximated by a GP-type distribution with one parameter which is the extreme value index. For reviews and references on fitting a GPD to exceedances over a sufficiently large threshold see (Castillo and Hadi, 1997; Beirlant et al., 1999, 2004; Thompson et al., 2009; Bermudez et al., 2010; MacDonald et al., 2011; Chikobvu et al., 2012; among others).

The rest of the paper is organized as follows. The GPD and GSP distribution are discussed in Section 2. Bayes estimation of the GPD and GSP distributions is discussed in Section 3. The data set is described in Section 4. The empirical results are discussed in Section 5. A detailed discussion of the significance of the empirical results is given in Section 6. The conclusion and policy implications are discussed in Section 7.

2. EVT and modelling of peak electricity daily changes

2.1. Generalized Pareto distribution

A peaks over threshold (POT) distribution is considered to model the observations above a sufficiently high threshold. The POT distribution considered here is the GPD with two parameters ξ, the shape parameter, also known as the Extreme Value Index (EVI) and σ, the scale parameter. Balkema and De Haan (1974) and Pickands (1975) showed that the distribution function of the excesses above a high threshold converges to a GPD as the threshold tends to the right endpoint.

Let $Y_1, Y_2, \ldots, Y_n$ be a sequence of daily increases in peak demand. The increase in peak demand is relative to the previous day. In order to extract upper extremes from this sequence we take the exceedances over a predetermined high threshold τ. The distribution function of the GPD is given in the following equation:

$$W_{\xi,\sigma}(y) = \begin{cases} 1-\left(1+\dfrac{\xi(y-\tau)}{\sigma}\right)^{-1/\xi} & \text{if } \xi>0, \quad y-\tau>0 \\ 1-\exp\left(-\dfrac{y-\tau}{\sigma}\right) & \text{if} \xi=0, \quad y-\tau>0 \\ 1-\left(1+\dfrac{\xi(y-\tau)}{\sigma}\right)^{-1/\xi} & \text{if } \xi<0, \quad 0<y-\tau<-\dfrac{\sigma}{\xi} \end{cases} \quad (1)$$

If $\xi>0$ then $W_{\xi,\sigma}(y)$ belongs to the heavy-tailed distributions such as Pareto, Student t, Cauchy, loggamma and Frechet whose tails decay like power functions. If $\xi=0$ then $W_{\xi,\sigma}(y)$ belongs to the Gumbel, normal, exponential, gamma and lognormal whose tails decay exponentially. If $\xi<0$, $W_{\xi,\sigma}(y)$ belongs to the uniform and beta distributions. The survival function of the GPD is given in equation:

$$P(Y>y|\tau) = \begin{cases} \left(1+\dfrac{\xi(y-\tau)}{\sigma}\right)^{-1/\xi} & \text{if } \xi>0, \quad y-\tau>0 \\ \exp\left(-\dfrac{y-\tau}{\sigma}\right) & \text{if } \xi=0, \quad y-\tau>0 \\ \left(1+\dfrac{\xi(y-\tau)}{\sigma}\right)^{-1/\xi} & \text{if } \xi<0, \quad 0<y-\tau<-\dfrac{\sigma}{\xi} \end{cases} \quad (2)$$

2.2. Generalized single Pareto distribution

In Verster and De Waal (2011), it is given that above a reasonably high threshold, τ, the tail of a Generalized Burr Gamma (GBG) distribution can be approximated by a GP-type distribution. The GP-type distribution, also a POT distribution, is an approximation of the GPD with the advantage of having only one parameter. The distribution and survival functions of the GP-type distribution which we also refer to as GSP distribution with shape parameter η (also known as the extreme value index (EVI)) are given as follows:

$$W_{\eta}(y) = 1-\left\{1+\frac{\eta}{1+\tau\eta}(y-\tau)\right\}^{-1/\eta}, \quad \eta\neq 0, \quad y>\tau \quad (3)$$

$$P(Y>y|\tau) = \left\{1+\frac{\eta}{1+\tau\eta}(y-\tau)\right\}^{-1/\eta}, \quad \eta\neq 0, y>\tau \quad (4)$$

3. Bayes estimation

3.1. Bayes estimation of the GPD

The two parameters are estimated jointly by considering a Bayesian approach. The joint posterior distribution of ξ and σ is given as follows:

$$\pi(\sigma,\xi|y) \propto \prod_{i=1}^{N_\tau} \frac{1}{\sigma}\left[1+\frac{\xi(y_i-\tau)}{\sigma}\right]^{-1/\xi-1} \pi(\sigma,\xi) \quad (5)$$

where $\pi(\sigma,\xi)\propto(1/\sigma)e^{-\xi}$ is the maximal data information (MDI) prior (Zellner, 1977) and N_τ is the number of observations above the threshold.

The two parameters are estimated by simulating a large number of σ's and ξ's values from the posterior distribution and taking the mean of the simulated values to obtain estimates. To simulate a set of (σ,ξ)'s from the posterior we make use of the Gibbs sampling method by simulating alternatively σ from its conditional density function given a fixed ξ. The parameter ξ is then simulated from its conditional density given the selected σ. This process is repeated a large number of times. Future posterior predictive tail probabilities of a future observation, Y_0, can be predicted through the following posterior predictive density:

$$P(Y_0>y_0|y,\tau) \propto \int\int \pi(\xi,\sigma|y)\left\{1+\frac{\xi}{\sigma}(y_0-\tau)\right\}^{-1/\xi} d\sigma\, d\xi, \quad -\infty<\xi<\infty \quad (6)$$

Eq. (6) cannot be computed analytically, but can be approximated easily by simulation. We simulate a large number of ξ's and σ's from the posterior distribution (Eq. (5)) which are then substituted into Eq. (6). The average over all the tail probabilities is then used to estimate the posterior predictive tail probability.

3.2. Bayes estimation of the GSP distribution

The tail index parameter η is estimated by considering a Bayesian approach and obtaining the posterior distribution of η. The posterior distribution of η is

$$\pi(\eta|y) \propto \prod_{i=1}^{N_\tau} \frac{1}{1+\eta\tau}\left[1+\frac{\eta(y_i-\tau)}{1+\eta\tau}\right]^{-1/\eta-1} \pi(\eta) \quad (7)$$

where $\pi(\eta)\propto e^{-\eta}/(1+\eta\tau)$ is the MDI prior (Zellner, 1977) and N_τ is the number of observations above the threshold. The mode of