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Inversion copulas from nonlinear state space models with an application to inflation forecasting

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ABSTRACT

We propose the construction of copulas through the inversion of nonlinear state space models. These copulas allow for new time series models that have the same serial dependence structure as a state space model, but with an arbitrary marginal distribution, and flexible density forecasts. We examine the time series properties of the copulas, outline serial dependence measures, and estimate the models using likelihood-based methods. Copulas constructed from three example state space models are considered: a stochastic volatility model with an unobserved component, a Markov switching autoregression, and a Gaussian linear unobserved component model. We show that all three inversion copulas with flexible margins improve the fit and density forecasts of quarterly U.S. broad inflation and electricity inflation.

1. Introduction

Parametric copulas constructed through the inversion of a latent multivariate distribution (Nelsen, 2006, Section 3.1) are popular for the analysis of high-dimensional dependence. For example, Gaussian (Song, 2000), *t* (Embrechts, McNeil, & Straumann, 2001) and skew-*t* (Demarta & McNeil, 2005; Smith, Gan, & Kohn, 2012) distributions have all been used to form such 'inversion copulas'. More recently, Oh and Patton (2017) suggested the employment of distributions formed by means of marginalization over a small number of latent factors. However, the copulas constructed from these distributions cannot capture the serial dependence exhibited by many time series accurately. As an alternative, we instead propose a broad new class of inversion copulas that are formed by inverting parametric nonlinear state space models. Even though the dimensions of such copulas are high, they are parsimonious because their parameters are those of the underlying latent state space model. The copulas also have the same serial dependence structure as the state space model. However, when combined with an arbitrary marginal distribution for the data, such copulas allow for the construction of new time series models. These models allow for substantially more flexible density forecasts than the underlying state space models themselves, because the latter typically have rigid margins that are often not consistent with those observed empirically.

When the latent state space model is non-stationary, the resulting copula model for the data is likewise, but with time-invariant univariate margins. Alternatively, when the state space model is stationary, so is the resulting copula model, and we focus on this case here. When the state space model is Gaussian and linear, the resulting inversion copula is a Gaussian copula (Song, 2000) with a closed form likelihood. However, in general, the likelihood function of a nonlinear state space model cannot be expressed in closed form, and similarly, neither can the density of the corresponding inversion copula. Nevertheless, we show how existing techniques for computing the likelihood of such state space models can also be used to compute the copula densities. We also provide an efficient spline approximation method for computing the marginal density

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and quantile function of the state space model. These are the most computationally demanding aspects of evaluating the copula density for time series data. We outline in detail how Bayesian techniques can be used to compute posterior estimates of the copula model parameters. A Markov chain Monte Carlo (MCMC) sampler is used, where the existing methods for sampling the states of a state space model efficiently can be employed directly. We also study the time series properties of the copula models, and show how to compute measures of serial dependence, as well as to construct density forecasts. We show that the density forecasts from the copula model reflect the nature of the empirical data distribution better than those of their state space model counterparts in real world applications.

Copula models are used extensively for modeling cross-sectional dependence, including between multiple time series; see Patton (2012) for a review. However, their use for capturing serial dependence has been much more limited. Beare (2010), Frees and Wang (2006), Joe (1997, pp. 243–280), Lambert and Vandenhende (2002), Loaiza-Maya, Smith, and Maneesoonthorn (forthcoming) and Smith, Min, Almeida, and Czado (2010) use Archimedean, elliptical or decomposable vine copulas to capture serial dependence in univariate time series. While the likelihood is available in closed form for these copulas, they cannot capture as wide a range of serial dependence structures as that captured by the inversion copulas proposed here. Moreover, the proposed inversion copulas are simple to specify and often more parsimonious, and can be easier to estimate than the copulas used previously.

Recently, copulas with time-varying parameters have proven popular for the analysis of multivariate time series data; see for example Almeida and Czado (2012), Creal and Tsay (2015), De Lira Salvatierra and Patton (2015) and Hafner and Manner (2012). However, these authors use copulas to account for the (conditional) cross-sectional dependence as per Patton (2006), which is completely different to our objective of constructing a T-dimensional copula for serial dependence. Semi- and nonparametric copula functions (Kauermann, Schellhase, & Ruppert, 2013) can also be used to model serial dependence. However, such an approach is better suited to longitudinal data, where there are repeated observations of the time series.

We highlight the broad range of new copulas that can be formed using our approach by considering three in detail. These are formed by the inversion of three stationary latent state space models that are popular in forecasting macroeconomic time series. The first is a stochastic volatility model with an unobserved first order autoregressive mean component. The second is a Markov switching first order autoregression. The third is a Gaussian unobserved component model, where the unobserved component follows a pth order autoregression. When forming an inversion copula, all characteristics (including moments) of the marginal distribution of the state space model are lost, leaving the parameters potentially unidentified. For each of the three inversion copulas that we study in detail, we solve this problem by imposing constraints on the parameter space. We show how to implement the MCMC sampling scheme, where the states are generated using existing methods and the parameters are drawn efficiently from constrained distributions. In an empirical setting, we also show how to estimate the copula parameters using maximum likelihood.

To show that using an inversion copula with a flexible margin can improve the forecast density accuracy substantially relative to employing the state space model directly, we use it to model and forecast quarterly U.S. broad inflation and electricity inflation. This is a long-standing problem regarding which there is a large body of literature (Faust & Wright, 2013). A wide range of univariate time series models have been used previously, including the three state space models examined here. However, all three have marginal distributions that are inconsistent with that observed empirically for inflation, as the latter exhibits a strong positive skew and heavy tails. Moreover, the predictive distributions from the state space models are all either exactly or approximately symmetric, which places an excessively high probability on severe deflation. In comparison, the inversion copula models employ the same serial dependence structures as the latent state space models, but also incorporate much more accurate asymmetric marginal distributions. We show that this not only improves the fit of the time series models, but also increases the accuracy of the one-quarter-ahead density forecasts significantly.

The rest of the paper is organized as follows. Section 2 begins by defining a time series copula model, then outlines the construction of an inversion copula from a nonlinear state space model. The special case of a Gaussian linear state space model is considered separately. We then discuss estimation, time series properties, measures of serial dependence and prediction. Section 3 provides a detailed discussion of the three inversion copulas that we examine, while Section 4 presents the analyses of the U.S. broad inflation and electricity inflation. Section 5 then concludes. An online supplement provides both a simulation study that verifies the proposed methodology in a controlled setting and supplementary figures for the U.S. electricity inflation application.

2. Time series copula models

Consider a discrete-time stochastic process $\{Y_t\}_{t=1}^T$, with a time series of observed values $\boldsymbol{y} = (y_1, \ldots, y_T)$. Then, a copula model decomposes its joint distribution function as

$$F_Y(\boldsymbol{y}) = C(\boldsymbol{u}), \tag{2.1}$$

where $\boldsymbol{u} = (u_1, \ldots, u_T)$, $u_t = G(y_t)$, and G is the marginal distribution function of Y_t, which we assume to be time-invariant. The function C is a T-dimensional copula function (Nelsen, 2006, p. 45) that captures all of the serial dependence in the data. All marginal features of the data are captured by G, which can be modeled separately, and either parametrically or non-parametrically. While Eq. (2.1) applies equally to continuous and discrete-valued time series data, we focus here on the former, where the density is

$$f_Y(\boldsymbol{y}) = \frac{d}{d\boldsymbol{y}} F_Y(\boldsymbol{y}) = c(\boldsymbol{u}) \prod_{t=1}^{T} g(y_t)\,. \tag{2.2}$$