International Journal of Forecasting 34 (2018) 276–287

Contents lists available at ScienceDirect

International Journal of Forecasting

journal homepage: www.elsevier.com/locate/ijforecast

Forecasting realized variance measures using time-varying coefficient models

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ARTICLE INFO

Keywords:
Volatility forecasting
Realized volatility
Measurement error
State space model

ABSTRACT

This paper considers the problem of forecasting realized variance measures. These measures are highly persistent estimates of the underlying integrated variance, but are also noisy. Bollerslev, Patton and Quaedvlieg (2016), Journal of Econometrics 192(1), 1–18 exploited this so as to extend the commonly used heterogeneous autoregressive (HAR) by letting the model parameters vary over time depending on the estimated measurement error variances. We propose an alternative specification that allows the autoregressive parameters of HAR models to be driven by a latent Gaussian autoregressive process that may also depend on the estimated measurement error variance. The model parameters are estimated by maximum likelihood using the Kalman filter. Our empirical analysis considers the realized variances of 40 stocks from the S&P 500. Our model based on log variances shows the best overall performance and generates superior forecasts both in terms of a range of different loss functions and for various subsamples of the forecasting period.

1. Introduction

Since accurate forecasts of asset volatility are crucial for option pricing, portfolio allocation and risk management, research has been investigating volatility modeling for over thirty years. The early models were the observation-driven class of GARCH models (Bollerslev, 1986; Engle, 1982) and the parameter driven class of stochastic volatility models (Taylor, 1982, 1986), both of which are typically applied to daily or weekly data. The increasing availability of high frequency data offers an alternative approach to estimating and forecasting the latent volatility process. Models based on lower frequency returns have (partially) lost their appeal, since they are not able to exploit the information available in the data fully.

In order to make intraday data applicable for estimating the true integrated variance (IV), Andersen and Bollerslev (1998) suggested estimating the asset volatility as the sum of squared intraday returns. The resulting realized variance (RV) is a consistent estimator for the IV as the sampling frequency goes to zero. The asymptotic theory for the realized volatility measure was derived by Barndorff-Nielsen and Shephard (2002). More sophisticated realized measures for estimating the integrated variance in the presence of jumps, microstructure noise or overnight returns have also been suggested in the literature. Prominent examples include the jump-robust bipower-variation of Barndorff-Nielsen and Shephard (2004), the subsampled realized variance of Zhang, Mykland, and Aït-Sahalia (2005) and the realized kernel of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2008). Nevertheless, Liu, Patton, and Sheppard (2015) showed that the standard RV estimator based on 5-min returns is difficult to beat, and it is still applied commonly in many applications.

The typical approach to modeling and forecasting the volatility is to treat realized variance measures as the true variance and apply reduced form econometric models. RV measures have been shown to be characterized by strong

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https://doi.org/10.1016/j.ijforecast.2017.12.005
0169-2070/

persistence, which must be taken into account when specifying an appropriate model. Andersen, Bollerslev, Diebold, and Labys (2003) propose that this persistence be modeled directly as a fractionally integrated process. Since the estimation of ARFIMA processes is cumbersome, the cascade model of Corsi (2009) has become the workhorse for modeling the long-memory of realized measures. The so-called heterogeneous autoregressive (*HAR*) model generates the persistence as the sum of three autoregressive components that reflect the investment horizons of different types of investors, namely the daily, weekly and monthly horizons. Since the *HAR* can be written as a restricted AR(20) model, parameter estimation using ordinary least squares is straightforward. Variance forecasts based on high frequency measures are superior to those based on GARCH or SV models fitted for daily returns, as was shown by Engle (2002) and Koopman, Jungbacker, and Hol (2005), for example. Furthermore, augmenting GARCH and SV models with RV measures based on high frequency data leads to improvements in both model fit and forecasting performance; see for example Engle and Gallo (2006), Hansen and Lunde (2012) and Shephard and Sheppard (2010) for observation-driven models and Dobrev and Szerszen (2010), Koopman and Scharth (2013) and Takahashi, Omori, and Watanabe (2009) for extended stochastic volatility models.

In addition to long memory, RV measures have a second feature that is relevant for the modeling and forecasting of volatilities that has mostly been neglected in the literature until recently, namely that the realized variance measures the integrated variance with an error as long as the sampling frequency is nonzero. Relying on the asymptotic distribution theory of Barndorff-Nielsen and Shephard (2002), Bollerslev, Patton, and Quaedvlieg (2016a) show how this heteroscedastic error translates into an attenuation bias, with the OLS estimate being attenuated with the average value of the measurement error variance. This implies that constant AR parameters are not optimal for forecasting. Bollerslev et al. suggest allowing for time-varying parameters in the *HAR* model. The time variation is driven by the variance of the measurement error of the realized variance, estimated by the realized quarticity, which results in forecasts that are superior to those of the basic *HAR* model. Their empirical results show that their resulting *HARQ* model also has a better forecasting performance than alternative *HAR*-type models, such as the *HAR* with jumps, the continuous *HAR* of Andersen, Bollerslev, and Diebold (2007) and the semivariance *HAR* of Patton and Sheppard (2015). Since the approach of Bollerslev et al. (2016a) models the *HAR* coefficients as a function of the realized quarticity, in principle the same approach can also be implemented for different variations of *HAR* models. Furthermore, the authors demonstrate that their approach is robust to the choice of the realized variance and quarticity estimators.

This paper's contribution is to propose an alternative model for forecasting realized volatility measures that exploits the potential presence of measurement errors. Our model is also based on the *HAR* model, but the first-order autoregressive coefficient is specified to be a latent Gaussian AR(1) process. The intuition behind this model is as follows: in the situation of heteroscedastic measurement errors, optimal forecasts are based on models with time-varying parameters. Since the realized quarticity is only a noisy measure of the variance of the measurement error, we propose to approximate the dynamics of the *HARQ* model by assuming latent AR(1) coefficients as a more robust alternative. The model parameters are estimated by maximum likelihood using a standard Kalman filter. Even though this basic specification does not exploit the realized quarticity as an estimate of the measurement error variance, it is still able to produce forecasts that are superior to those generated by the *HAR* and *HARQ* models. As an extension, we consider models that combine the state space specification with the idea of Bollerslev et al. (2016a). First, we augment the state equation for the time-varying parameter with a realized quarticity. A variant of this extension contains an indicator such that the realized quarticity is effective only when it exceeds the 99% quantile of its in-sample values. Thus, the model uses this additional information only when the measurement error variance is exceptionally large. Second, we study a model that combines the time-varying parameters of the *HARQ* model and our state space model. Furthermore, we consider the *HAR* model in terms of the natural logarithm in both the basic and state space forms, an approach that results in the most promising empirical results.

In our empirical application, we use a large dataset of 40 stocks from the S&P 500 index over a period of 15 years. We compare the in-sample fits and forecasting performances of our models for realized variances based on 5-min returns. Furthermore, we consider subsamples of the forecasting period covering periods of high and low volatility. Our state space model based on log volatilities shows the best performance of all models compared, and consistently outperforms the *HARQ* models for forecasting the volatility.

The remainder of the paper is structured as follows. Section 2 discusses the theoretical framework, reviews existing approaches and introduces our model. Section 3 compares the competing models in terms of both model fits and forecasting performances, and Section 4 concludes.

2. Methodology

2.1. Setup and existing approaches

Consider an asset whose price process P_t is given by the stochastic differential equation

$$d\log(P_t) = \mu_t dt + \sigma_t dW_t, \tag{1}$$

where μ_t denotes the drift, σ_t the instantaneous volatility and W_t a standard Brownian motion. The integrated variance for day t is then defined as

$$IV_t = \int_{t-1}^{t} \sigma_s^2 ds. \tag{2}$$

Let $r_{t,i} = \log(P_{t-1+i\Delta}) - \log(P_{t-1+(i-1)\Delta})$ be the ith intraday return over a period of length Δ and assume that $M = 1/\Delta$ intraday returns are available. A consistent estimator for the integrated variance as $\Delta \to 0$, assuming that no jumps