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A prediction interval for a function-valued forecast model: Application to load forecasting

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ABSTRACT

Starting from the information contained in the shape of the load curves, we propose a flexible nonparametric function-valued forecast model called KWF (*Kernel + Wavelet + Functional*) that is well suited to the handling of nonstationary series. The predictor can be seen as a weighted average of the futures of past situations, where the weights increase with the similarity between the past situations and the actual one. In addition, this strategy also provides simultaneous predictions at multiple horizons. These weights induce a probability distribution that can be used to produce bootstrap pseudo predictions. Prediction intervals are then constructed after obtaining the corresponding bootstrap pseudo prediction residuals. We develop two propositions following the KWF strategy directly, and compare it to two alternative methods that arise from proposals by econometricians. The latter involve the construction of simultaneous prediction intervals using multiple comparison corrections through the control of the family-wise error (FWE) or the false discovery rate. Alternatively, such prediction intervals can be constructed by bootstrapping joint probability regions. In this work, we propose to obtain prediction intervals for the KWF model that are valid simultaneously for the H prediction horizons that correspond to the relevant path forecasts, making a connection between functional time series and the econometricians' framework.

1. Introduction

The recent literature proposes a range of methods for short-term electricity demand forecasting following various different types of approaches, such as time series analysis, machine learning, regression, and similarity searches. Some examples of such papers, restricting our attention to studies involving French electricity consumption, are as follows. Taylor (2010) uses an exponential smoothing method to take into account the structure of seasonality; while Dordonnat, Koopman, Ooms, Dessertaine, and Collet (2008) and Dordonnat, Koopman, and Ooms (2012) propose a state space model that allows changes in the relationship between exogenous factors (mainly temperature) and the demand for electricity to be tracked. Bruhns, Deurveilher, and Roy (2005) model this dependence using a nonlinear regression on the temperature, depending on the month, day of the week and time of day. A nonparametric version of this strategy was proposed recently by Pierrot and Goude (2011), and a Bayesian ap-

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proach was provided by Launay, Philippe, and Lamarche (2012). Among the machine learning methods, Devaine, Goude, and Stoltz (2011) propose a mixture of online predictors for obtaining forecasts that are adapted to nonstationarity. The last group of models, based on similarity searches, is an alternative to modeling the dependence structure of seasonal cycles. The basic idea is that similar cases in the past have similar future consequences. For example, Poggi (1994) divides the trajectory of the electricity consumption into one-day blocks, then, using dissimilarity measures, finds blocks in the past that are similar to the last observed block, and builds a weight vector. Finally, forecasts of the next two days are obtained via a weighted average of the most similar days, where the weights are given by the weight vector. From a statistical point of view, the model is an estimate of the regression function, using the kernel method, of the last block against all previous blocks. Antoniadis, Paparoditis, and Sapatinas (2006) extend this model to the case of stationary functional random variables. However, the hypothesis of stationarity may fail in the context of the French electricity demand, as an evolving mean level and the existence of groups that may be seen as classes of stationarity are to be considered. We explore some corrections to take into account these two main nonstationary features. Let us now be a little more precise.

Electricity load experts naturally consider daily demand data as time functions called load curves. In a recent paper, Shang (2013) used a functional time series approach for forecasting the short-term electricity demand. The approach is illustrated using the half-hourly electricity demand from Monday to Sunday in South Australia. The strategy also involves considering a seasonal univariate time series as a time series of curves, then reducing the dimensionality of the curves by applying a functional principal component analysis; finally, following Shang and Hyndman (2011), the principal component scores are forecast using univariate ARIMA models. In addition, since the data points in the daily electricity demand are observed sequentially, a forecast updating method based on the nonparametric bootstrap approach is proposed in order to improve the accuracy of point forecasts. Relative to this strategy, the scheme that we propose handles the forecasting problem in a more functional way, avoiding the hour by hour processing, and considers a more flexible way of constructing the distribution leading to the prediction interval.

The shapes of the curves exhibit rich information about factors such as the calendar day type, the meteorological conditions, or the existence of special electricity tariffs. Using the information contained in the shape of the load curves, Antoniadis, Brossat, Cugliari, and Poggi (2012) proposed a flexible nonparametric function-valued forecast model called KWF (*Kernel + Wavelet + Functional*) that is well suited to the handling of nonstationary series. The predictor can be seen as a weighted average of the futures of past situations, where the weights increase with the similarity between the past situations and the actual one. In addition, this strategy provides simultaneous predictions for multiple horizons.

Moreover, the weights from the KWF model induce a probability distribution that can be used to produce bootstrap pseudo predictions. Antoniadis, Brossat, Cugliari, and Poggi (2014) constructed prediction intervals after obtaining the corresponding bootstrap pseudo prediction residuals. However, when applied in the electrical context, the intervals obtained are not completely satisfactory. First, the theoretical results only provide a pointwise coverage. Second, the dependency structure of the curves is (almost) not used.

Interestingly, econometricians have worked on a similar framework. Let $(y_t)_{t\in\mathbb{Z}}$ be a time series observed at time $t = 1, \ldots, T$. Just after observing y_T, we want to produce a path forecast, i.e., construct a predictor of the future H values of the series $\mathbf{y}_H = (y_{t+1}, \ldots, y_{t+H})'$ and a simultaneous prediction interval (PI) for the path, i.e., to construct a set $A \subset \mathbb{R}^H$ such that $P(\mathbf{y}_H \in A) \geq 1 - \alpha$, for some small $\alpha \in [0, 1]$.

The construction of simultaneous PI (i.e., intervals for a random variable) follows the guidelines for the construction of simultaneous confidence intervals (i.e., intervals for a parameter). They can be constructed marginally for each prediction horizon or simultaneously using multiple comparison corrections through the control of the family-wise error (FWE), using for example the Bonferroni correction, or through the control of the false discovery rate (Benjamini, Madar, & Stark, 2013).

This subject has been studied recently by various econometricians who are interested in path forecasts where bootstrap pseudo predictions can be produced. Staszewska (2007) proposes a heuristic method for eliminating any extreme bootstrap trajectories, then constructs the PI as the convex hull of the remaining trajectories. The PI can also be constructed by estimating a joint probability region under assumptions that can be quite strong. Jordà and Marcellino (2010) constructed this region by means of an asymptotic normal approximation. On the other hand, Wolf and Wunderli (2013) constructed the joint probability region using a bootstrap strategy where the calibration is done by controlling the multiple comparisons by means of a generalized notion of FWE (k-FWE). In this way, all but a small number k of horizons are guaranteed to be covered. Recently, Delattre and Roquain (2013) proved a theoretical result about the validity of k-FWE. Some additional details of these methods are given in Section 2.3.

In this work, we propose to obtain PI for the KWF model that are valid simultaneously for $y_{t+1}, \ldots, y_{t+H}$, the H prediction horizons that correspond to the relevant path forecast. This shows the connection between functional time series and the econometricians' framework.

Two final studies are of interest here. Firstly, Petiau (2009) presents a method for obtaining prediction intervals for load forecasts, based on the calculation of empirical quantiles of the distribution of the relative forecast error observed in the past. An a priori classification of days for which the load forecasting is difficult and days for which it is easier is used, together with the hour within the day of the forecast, to take into account the past error forecasts that are included in the distribution calculation. The scheme is applied either to the whole French electrical network or to each of the seven French regional networks individually. Relative to this strategy, the scheme that we propose handles the forecasting problem in a functional

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