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Comparison of methods for constructing joint confidence bands for impulse response functions

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ABSTRACT

In vector autoregressive analyses, confidence intervals for individual impulse responses are typically reported in order to indicate the sampling uncertainty in the estimation results. Various methods are reviewed, and a new method for the construction of joint confidence bands, given a prespecified coverage level, for the impulse responses at all horizons considered simultaneously, is proposed. The methods are compared in a simulation experiment, and recommendations for empirical work are provided.

1. Introduction

In vector autoregressive (VAR) analyses, impulse responses are commonly used for investigating the effects of shocks on the system. In practice, because the impulse responses are functions of the VAR parameters, they have to be estimated. Estimation uncertainty is usually indicated by showing confidence intervals around the individual impulse response coefficients. Asymptotic, bootstrap and Bayesian methods are typically used for setting up such intervals (see, e.g., Lütkepohl, 2005).

Despite this practice of reporting the estimation uncertainty for individual impulse response coefficients, economists are often interested in the response of a certain variable to a specific shock over a longer propagation horizon. For example, in a standard real business cycle (RBC) model, a technology shock is expected to increase hours worked in the long-run, that is, for a number of future periods (see, e.g., Galí, 1999, for an empirical investigation of this issue). Similarly, a contractionary monetary policy shock is expected to reduce the price level and bring down inflation (e.g., Uhlig, 2005). When responses over several periods are of interest, it is desirable to have confidence bands for impulse response functions rather than confidence intervals for individual impulse response coefficients.

If individual confidence intervals for a given confidence level are constructed around the impulse response coefficients for each response horizon separately, there is no guarantee that the overall coverage level for all impulse responses of one variable will correspond to the prespecified confidence level. In other words, the probability of the band containing the true impulse response function of a specific variable will generally not be $1 - \gamma$ if the confidence band is constructed as the union of individual $(1 - \gamma) \times 100\%$ confidence intervals. Hence, it is desirable to construct confidence bands with an overall prespecified coverage probability. A range of suitable methods are reviewed in this study and a new proposal is considered. A simulation experiment is used to compare the methods, and recommendations for applied work are given. Our criteria for assessing the bands are the coverage level and the width of the confidence band. While different measures of width are conceivable, in this study we calculate it as the sum of the widths of all individual intervals.

This is not the first study to consider the problem of constructing confidence bands for impulse responses. For

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example, Inoue and Kilian (2013) and Sims and Zha (1999) propose methods based on Bayesian principles. In this study, we will remain within a classical framework where one could use, for example, the Bonferroni inequality for constructing confidence bands with a joint coverage level at least as large as the desired one. The drawback of this method is that it may deliver very conservative bands that provide much larger coverages than desired, and consequently, are unnecessarily wide. Therefore, we propose a strategy for reducing the bands by adjusting the Bonferroni bands. Another proposal was made by Jordà (2009). He constructed the bands on the basis of so-called Scheffé bounds. Unfortunately, though, the underlying inequalities are only approximate, and may fail to deliver correct coverage levels even under ideal conditions, as was argued convincingly by Wolf and Wunderli (2012) in the context of constructing joint forecast bands. In the context of constructing confidence bands for impulse responses, the simulation evidence from Kilian and Kim (2009) points in the same direction. Yet another approach was proposed by Staszewska (2007), who used numerical search methods to find the smallest possible confidence bands for a given coverage level. The disadvantage of this is that it requires a rather substantial computational effort. Moreover, no general results are available to show that the desired coverage level is actually obtained at least asymptotically. All of these proposals will be compared in a simulation experiment.

In the present study, we consider bands for the impulse response functions of individual variables; that is, we consider confidence bands for the response of an individual variable to a specific shock. This approach is in line with the bands proposed and investigated in most of the related literature (e.g., Staszewska, 2007). In contrast, Inoue and Kilian (2013) point out that it may be appropriate to consider the full uncertainty in all impulse response functions jointly. Although some of the methods discussed below can be extended in that direction, we focus on bands for individual impulse response functions because they may be more relevant from a practical point of view.

Bands with given coverage levels are also of interest in computing forecast paths over a number of horizons. The construction of bands around path forecasts has been considered, for instance, by Jordà and Marcellino (2010), Staszewska-Bystrova (2011), Staszewska-Bystrova and Winker (2013) and Wolf and Wunderli (2012). Since impulse responses are conditional forecasts, there is an obvious relationship with the forecast literature, which we will draw on by adapting the method proposed by Wolf and Wunderli (2012) to our framework of constructing confidence bands around impulse responses. The difference between this and the literature on path forecasts is that there are two components of uncertainty attached to forecasts of specific variables, even if the data generation process (DGP) is known apart from its parameters: the intrinsic uncertainty from the DGP and the estimation uncertainty obtained from using estimated instead of true parameters. In contrast, since impulse responses are conditional forecasts that consider only the marginal effect of a specific shock for a given process, only the estimation uncertainty is relevant in the context of evaluating the impulse responses if the correct model is used. Of course, in practice, there is the usual uncertainty about the DGP in both types of analysis. In any case, our results are also of interest for constructing bands around multiple-horizon forecasts for specific variables, although we focus on the impulse response context.

The remainder of the study is organized as follows. In Section 2, the model setup is presented. Section 3 reviews the methods for constructing joint confidence bands for impulse responses, and a simulation comparison is discussed in Section 4. An illustrative example is given in Section 5, and Section 6 concludes. A number of technical details can be found in the Appendix.

2. Model setup

A standard reduced-form VAR setup is used, with the variables $y_t = (y_{1t}, \ldots, y_{Kt})'$ being generated by a K-dimensional VAR(p) process,

$$y_t = \nu + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t. \tag{2.1}$$

The A_i ($i = 1, \ldots, p$) are ($K \times K$) parameter matrices, and the error process $u_t = (u_{1t}, \ldots, u_{Kt})'$ is a K-dimensional zero mean white noise process with covariance matrix $\mathbb{E}(u_t u_t') = \Sigma_u$, that is, $u_t \sim (0, \Sigma_u)$. The K-dimensional intercept vector ν is the only deterministic term, because such terms are of limited relevance for the following arguments. Adding other terms such as linear trends or seasonal dummy variables would not change the substance of the argument, though, in practice, they need to be included as they are required for a good description of the data, of course.

In lag operator notation, the process in Eq. (2.1) can be written as

$$A(L)y_t = \nu + u_t \tag{2.2}$$

with $A(L) = I_K - A_1 L - \cdots - A_p L^p$. The process is *stable* if

$$\det A(z) = \det(I_K - A_1 z - \cdots - A_p z^p) \neq 0 \quad \text{for } z \in \mathbb{C},\ |z| \leq 1. \tag{2.3}$$

Structural shocks ε_t are obtained from the reduced-form errors by a linear transformation, $\varepsilon_t = B^{-1}u_t$, such that the structural shocks are instantaneously uncorrelated and have a variance of one. In other words, the ($K \times K$) transformation matrix B has to be such that $BB' = \Sigma_u$, and hence, $\varepsilon_t \sim (0, I_K)$. The matrix B is the matrix of impact effects of the shocks. It is not determined uniquely by the relationship $BB' = \Sigma_u$, but is assumed to be identified by a suitable set of further restrictions. These can be either exclusion restrictions on the impact effects or constraints for the long-run effects of the shocks. For stable processes, the impulse responses are just the coefficients of the moving average (MA) representation of y_t,

$$y_t = A(1)^{-1}\nu + A(L)^{-1}B\varepsilon_t = \mu + \sum_{i=0}^{\infty} \Phi_i \varepsilon_{t-i}, \tag{2.4}$$

where $\mu = A(1)^{-1}\nu$, $\Phi_0 = B$ and $\sum_{i=0}^{\infty} \Phi_i L^i = A(L)^{-1}B$. Hence, the impulse response coefficients $\Phi_i = \Phi_i(A_1, \ldots, A_p, B)$ are nonlinear functions of the reduced-form parameters and B (see, e.g., Lütkepohl, 2005, Chapter

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