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Pretesting for multi-step-ahead exchange rate forecasts with STAR models

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ABSTRACT

It is well known that a linear model may forecast better than a nonlinear one, even when the nonlinear model is consistent with the actual data-generating process. Moreover, forecasting with nonlinear models can be quite programming-intensive, as multi-step-ahead forecasts need to be simulated. We propose a simple pretest to help determine whether it is worthwhile to forecast a series using a *STAR* model. In particular, we extend Teräsvirta's in-sample test for *LSTAR* and *ESTAR* behavior to multi-step-ahead out-of-sample forecasts. We apply our pretest to the real exchange rates of various OECD countries. When the test strongly rejects the null of linearity, a nonlinear model clearly outperforms a linear one in terms of multi-step-ahead forecasting accuracies (i.e., lower mean absolute percentage errors). However, when it fails to reject the null or does so only mildly, a direct approach based on an autoregressive model yields forecasts that are slightly superior to those generated from a logistic model. We also find that, when the proposed test strongly rejects the null of linearity, the "direct" method of forecasting and the bootstrap predictor yield similar performances, with the latter outperforming in terms of lower mean absolute percentage errors.

1. Introduction

Teräsvirta (2006, chap. 8) summarizes much of the research indicating that a linear model may forecast better than a nonlinear one, even when the nonlinear model is consistent with the actual data-generating process. For example, Montgomery, Zarnowitz, Tsay, and Tiao (1998) show that a nonlinear model may forecast better than a linear one in some regimes but not in others (e.g., recessions but not expansions). Similarly, Dacco and Satchell (1999) show that a regime-switching model may have a poor forecasting performance relative to a linear model, as a result of misclassifying observations. The point is that a superior in-sample performance of a nonlinear model may not translate into a superior out-of-sample performance.

White's (2006) survey article details the "potentially serious practical challenges" that are inherent in using a nonlinear model to make multi-step-ahead forecasts. He shows that such forecasts are computationally difficult to program, entail the possibility of overfitting, and are often difficult to interpret. In the same volume, Teräsvirta (2006, chap. 8) highlights the different methodological choices that can lead a researcher astray when using a nonlinear model to make multi-step-ahead forecasts. For example, forecasters need to select the appropriate model, decide whether to use direct or indirect methods, decide whether to use Monte Carlo or bootstrapped forecasts, and select the appropriate histories for their simulations. As time series forecasting competitions have shown (see Weigend & Gershenfeld, 1994), different well-trained researchers will

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often come up with very different multi-step-ahead forecasts using the same data set. In order to avoid many of these forecasting issues, we extend the nonlinearity tests of Luukkonen, Saikkonen, and Teräsvirta (1988), Teräsvirta (1994), and Tsay (1991). Specifically, we extrapolate their in-sample methodologies to develop a simple pretest to determine whether it is better to forecast a series using a smooth transition autoregressive (*STAR*) model or a linear model.

We focus our attention on *STAR* models because they are particularly well-suited for characterizing important macroeconomic variables such as the unemployment rate, the growth rate of industrial production, and exchange rates (see Marcellino, 2002, 2004, and Stock & Watson, 1998, among others). Moreover, as will be shown, the multi-step-ahead forecasts of the type of *STAR* models we consider have a *STAR* form. This greatly eases the specification problem, in that the form of the model does not change at various forecasting horizons. Nevertheless, testing for linearity directly is not straightforward, because of the unidentified parameters under the null (see Davies, 1987). However, in the context of *STAR* models, Teräsvirta (1994) shows how to circumvent this nuisance parameter problem. The solution consists of taking a Taylor series approximation of the nonlinear component (i.e., the transition function) of the *STAR* process. This paper adopts this approach with the aim of establishing whether the data-generating process appears to be nonlinear at several steps ahead.

Several papers, including those of Bec, Salem, and Carrasco (2004), Kapetanios, Shin, and Snell (2003), Kilian and Taylor (2003), Leybourne, Newbold, and Vougas (1998), Michael, Nobay, and Peel (1997), and Sollis, Leybourne, and Newbold (2002), suggest that the behavior of the real exchange rate can be characterized be some type of nonlinear adjustment. As such, it seems natural to apply our test to a set of real exchange rates from a variety of OECD countries. When our linearity test strongly rejects the null of a linear model, the forecasts from a nonlinear model have lower mean square prediction errors (*MSPE*s) than those from a linear model. Moreover, when the linearity test fails to reject the null of a linear model, the forecasts from the *STAR* model and its linear counterpart display similar *MSPE*s. We show that the nonlinear exchange rate models can also beat the benchmark random-walk forecasts.

The remainder of the paper is organized as follows. Section 2 serves as a brief literature review, in that it describes the logistic (*LSTAR*) and exponential (*ESTAR*) forms of the *STAR* model. Section 3 shows how we can generalize Teräsvirta's (1994) test for STAR behavior to multi-step-ahead forecasts. Section 4 performs several size and power exercises, to assess the small-sample properties of our pretest. Section 5 applies the test to the real exchange rates of many OECD countries and uses the results of the test to construct out-of-sample forecasts. We follow the lead of Lin and Granger (1994) and construct various out-of-sample forecasts using both the direct and iterative (recursive) forecasting methods. We find that the direct method has the advantage that no numerical integration and/or approximation is necessary, and the forecasts can be produced as in the one-step-ahead case. When our test indicates that the data generating process (*DGP*) is nonlinear, it turns out that, on average, iterated forecasts from a nonlinear model of the real exchange rates have lower *MAPE*s than those from a direct approach. In contrast, when our test indicates that the *DGP* is linear, the direct method results in lower root mean squared errors. Section 6 summarizes the key results.

2. The smooth transition regression model

As was detailed by Granger and Teräsvirta (1993) and Teräsvirta (2006, chap. 8), a univariate smooth transition autoregressive (*STAR*) model has the following representation:

$$y_{t+1} = \alpha w_t + \beta w_t G(\theta; y_{t-d}; c) + \varepsilon_{t+1},$$
$$t = 1, \dots, T, \tag{1}$$

where $\varepsilon_{t+1} \sim i.i.d.\ (0, \sigma^2)$, α, β, and c, are unknown parameters, $w_t = (y_t, \dots, y_{t-p})$, $d \geq 0$ is the delay parameter, and the transition function $G(\theta; y_{t-d}; c)$ is continuous and may be either odd or even.

One common formulation of the transition function is the logistic equation:

$$G(\theta; y_{t-d}; c) = [1 + \exp(-\theta(y_{t-d} - c))]^{-1}, \tag{2}$$

where c is the location (or threshold) parameter.

In Eq. (2), the θ parameter represents the slope of the transition function $G(\theta; y_{t-d}; c)$. Specifically, when $\theta = 0$, the transition function $G(\theta; y_{t-d}; c) \equiv 1/2$, so that the *LSTAR* model nests a linear model. Conversely, when $|\theta| \to \infty$, the *LSTAR* model approaches a threshold autoregressive model with two distinct regimes. The transition function is a bounded function of the transition variable y_{t-d} and is continuous everywhere in the parameter space for any value of y_{t-d}. Given that $G(\theta; y_{t-d}; c)$ is odd and monotonically increasing, whenever $|y_{t-d} - c|$ is large and $y_{t-d} < c$, y_{t+1} is effectively generated by the linear model:

$$y_{t+1} = \alpha w_t + \varepsilon_{t+1}, \quad t = 1, \dots, T. \tag{3}$$

If $|y_{t-d} - c|$ is large and $y_{t-d} > c$, y_{t+1} is effectively generated by:

$$y_{t+1} = (\alpha + \beta) w_t + \varepsilon_{t+1}, \quad t = 1, \dots, T. \tag{4}$$

Thus, the parameters of the autoregressive process change monotonically as a function of y_{t-d}. Such a model might be appropriate for the real exchange rate behavior if the speed of adjustment depends on whether the rate is appreciating or depreciating.

If $G(\theta; y_{t-d}; c)$ is assumed to be even, it is standard to use the exponential function to model the transition between two regimes. The *ESTAR* transition function can be written as:

$$G(\theta; y_{t-d}; c) = 1 - \exp[-\theta(y_{t-d} - c)^2]. \tag{5}$$

The parameters θ, c and d are defined as above. Similarly to the logistic case, the exponential function is bounded between 0 and 1, cases that correspond to $\theta = 0$ and $\theta \to \infty$, respectively. Eq. (1), with Eq. (5), has been employed specifically to test the empirical validity of the purchasing power parity (*PPP*) hypothesis (see Taylor & Sarno, 2002, for a review).

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