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Efficient estimation of forecast uncertainty based on recent forecast errors

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ABSTRACT

Multi-step-ahead forecasts of the forecast uncertainty of an individual forecaster are often based on the horizon-specific sample means of his recent squared forecast errors, where the number of past forecast errors available decreases one-to-one with the forecast horizon. In this paper, the efficiency gains from the joint estimation of forecast uncertainty for all horizons in such samples are investigated. If the forecast uncertainty is estimated by seemingly unrelated regressions, it turns out that the covariance matrix of the squared forecast errors does not have to be estimated, but simply needs to have a certain structure, which is a very useful property in small samples. Considering optimal and non-optimal forecasts, it is found that the efficiency gains can be substantial for longer horizons in small samples. The superior performance of the seemingly-unrelated-regressions approach is confirmed in several empirical applications.

1. Introduction

In recent years, many forecasting institutions have supplemented their point forecasts with measures of forecast uncertainty. That is, they forecast not only the central tendency, but also some measure of the dispersion of the forecast density, which is communicated, for example, by the width of fan charts. Examples of such institutions include the Federal Reserve, the European Central Bank (ECB), the Bank of England (BoE), the Bank of Canada, the International Monetary Fund, the Sveriges Riksbank, the United States Congressional Budget Office and the Deutsche Bundesbank.[1]

As was stated by Wallis (1989, p. 56), "Estimating the future margin of error is itself a forecasting problem". When investigating uncertainty forecasts, researchers typically start by considering a general forecasting model. Within this model, they identify different sources of forecast uncertainty, such as estimation uncertainty and the accumulation of future errors. Then, the uncertainty of the forecasts can be determined as the aggregate impact of these sources. Examples of this approach for assessing the forecast uncertainty can be found, for instance, in the work of Clements and Hendry (1998, Ch. 7) and Ericsson (2002).

However, as Wallis (1989, pp. 55–56) noted, "This approach is of little help to the practitioner. It neglects the contribution of the forecaster's subjective adjustments [...]. More fundamentally, the model's specification is uncertain. At any point in time competing models coexist, over time model specifications evolve, and there is no way of assessing this uncertainty. *Thus, the only practical indication of the likely margin of future error is provided by the past forecast errors*" (emphasis added). It should be noted that this approach, of course, has to rely on the assumption that the past forecast uncertainty is a good indicator of the future forecast uncertainty, which might not always be the case. Nevertheless, past forecast errors do indeed play a central role in the assessment of forecasting uncertainty for all of the forecasting institutions mentioned.[2]

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[1] The European Central Bank only reports a forecast range, so that actually only the forecast uncertainty is presented, not the exact central tendency. All of the institutions mentioned publish forecasts for several periods ahead.

[2] See Deutsche Bundesbank (2010, pp. 34–36) for a more comprehensive survey.

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There is a large body of literature on the estimation of forecast uncertainty in panel settings, where forecasts of various individual forecasters are available. Important contributions to this literature were provided by Davies and Lahiri (1995), Giordani and Söderlind (2003), Lahiri and Sheng (2010), Liu and Lahiri (2006) and Zarnowitz and Lambros (1987). In many cases, this branch of the literature addresses the question of how the (aggregate) forecast uncertainty is related to the forecasts provided by individual forecasters. The present work, however, is concerned with the estimation of the forecast uncertainty in the absence of a panel of forecasters.[3] In practice, this appears to be a very relevant issue, because most forecasting institutions base their assessments of forecast uncertainty on their own past forecast errors only. An important exception is the Federal Reserve, which uses the forecast errors of several private and government forecasters, as was documented by Reifschneider and Tulip (2007).

Surprisingly, issues concerning the estimation of the forecast uncertainty of an individual forecaster based only on his past forecast errors have hardly been investigated in the literature to date. A notable exception is provided by Williams and Goodman (1971).[4] The lack of research with respect to the calculation of forecast uncertainty from past forecast errors is most likely to be due to the fact that this calculation can be carried out in an extremely simple way. First, one collects all of the forecast errors for each forecast horizon. Then, one performs a suitable transformation on these errors, reflecting the measure of dispersion to be reported. Typically, this means taking either absolute values or squared values of the forecast errors. In this work, I will focus on squared errors. For each horizon, the sample mean of the squared errors is calculated, i.e., an ordinary least squares (OLS) regression of the squared errors on a constant is performed. This OLS estimation yields consistent estimates of the forecast uncertainty. It is apparently used by all of the institutions mentioned above.[5] However, since the forecast errors are correlated across horizons, this procedure is not efficient.

This inefficiency is particularly pronounced for longer forecast horizons in small samples, for two reasons. Firstly, the autocorrelation of the forecast errors typically increases with the forecast horizon, so that estimates for long horizons tend to be rather imprecise. Secondly, the number of forecast errors available often decreases with the horizon, due to the fact that, for the most recent forecasts, only the forecast errors for short horizons can be calculated, because realizations are only available for these horizons. If the frequency of forecast publications equals the frequency of the forecast variables, the number of available forecast errors decreases one-to-one with the forecast horizon. I will refer to such samples of forecast errors as samples of *recent* forecast errors. In practice, samples of recent forecast errors are frequently used to estimate the forecast uncertainty. These samples are usually present if a forecaster uses all forecast errors from the introduction of a new forecasting regime to the present.

These samples are often rather short, due to either recent changes in the forecasting regime, or the short history of the forecasting institution itself. For example, the ECB came into existence in 1999 and has been producing macroeconomic forecasts conditional on the interest rates expected by market participants instead of constant interest rates only since June 2006.[6] Also, several central banks have switched to inflation-targeting regimes over the last two decades, and therefore, often only a very few recent forecast errors are available.[7]

In this work, I consider an estimator based on seemingly unrelated regressions (SUR estimator). The small-sample efficiency gains of this estimator are investigated for samples of recent forecast errors in the case of optimal forecasts. It turns out that the SUR estimator has the surprising property that its projection matrix does not depend on the data-generating process, and therefore its projection matrix does not need to be estimated, but simply requires a certain structure. This is an intriguing property in small samples.

In practice, most forecasts are probably non-optimal. Monte Carlo studies show that the OLS estimator can sometimes be more efficient than the SUR estimator, but only if the forecasting model suffers from severe misspecification. However, the SUR estimator continues to yield better results in most cases studied, often even in cases of severe misspecification. The OLS estimator is less biased in the case of structural breaks, but the SUR estimator can be preferable nevertheless, due to its smaller variance.

Finally, I apply the SUR estimator to forecasts made by the BoE, the ECB, and Consensus Forecasts. The SUR estimator is found to deliver more stable estimates of the multi-step-ahead forecast uncertainty than the OLS estimator in almost all cases. This indicates that, in general, efficiency gains are obtained with the SUR estimator in empirical applications as well.

3 Using the terminology of Lahiri and Sheng (2010), this work is concerned with the estimation of the variance of the individual forecast error. It is not concerned with the decomposition of that variance into the uncertainty associated with common shocks and the variance of the idiosyncratic shocks.

4 If empirical forecast errors are not yet available, because the new forecasting regime has only recently come into existence, one might consider assessing the future forecast uncertainty by trying to find similar regimes for which empirical forecast errors have already been observed. Such an approach was proposed by Makridakis, Hogarth, and Gaba (2009).

5 Based on the estimated forecast uncertainty, prediction intervals covering a certain probability of the forecast density are calculated in many cases. These prediction intervals of course require distributional assumptions for the forecast errors.

6 See European Central Bank (2006, p. 75).

7 The small sample size can be a serious impediment to the publication of forecast uncertainty estimates based on past forecast errors. For example, the National Bank of Poland has been using estimates of the forecast uncertainty based on past forecast errors only since October 2008. Before, the estimates were purely model-based. The reason for this change in the methodology was the consideration that the sample of past forecast errors had finally become large enough. See National Bank of Poland (2008, p. 70).