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A robust optimization approach for itinerary planning with deadline

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ABSTRACT

We propose a robust optimization approach to address the itinerary planning problem with deadline in public transit networks. Given departure times at origins and deadlines at destinations, we help the travelers meet the deadlines as much as possible. Our model maximizes the size of the uncertainty set of arc travel times, while guaranteeing that the corresponding worst-case arrival time of itinerary would not exceed the deadline. We exploit the model's structure and develop efficient solution algorithms. We demonstrate in numerical studies that our approach can effectively mitigate the lateness and can solve real-world instances within one second.

1. Introduction

In urban bus networks, each headway-based bus line runs along a predetermined route and serves a sequence of bus stops. Using these bus services, travelers face the problem of determining the optimal itinerary from an origin to a destination. In the deterministic setting, an optimal itinerary determines the bus lines and the transfer stops for the travel with the minimum travel time. The *itinerary planning problem* (IPP), which is usually modeled as a variant of the *shortest path problem* (Dial, 1967), is a fundamental and important problem in public transit operations management. Several important problems in public transit research, such as transit assignment and transit network design, have IPPs integrated as subroutines, which underscores the importance of having effective and efficient algorithms for solving the IPP and its variants.

Uncertainty exists ubiquitously in travel times along bus lines and transfer waiting times at stops. In particular, the waiting time in congested bus networks is difficult to predict in the itinerary planning phase. However, most studies in the literature about the IPPs ignore uncertainty, which can potentially lead to tardiness in meeting a prescribed deadline. In terms of characterizing uncertainty, the decision theory distinguish the difference between *risk* and *ambiguity* (Knight, 1921); the former represents uncertainty by probability distributions, while the latter represents uncertainty via an *uncertainty set* of outcomes without associating them with any probability distribution. Under the paradigm of *stochastic programming*, risk is assumed and a traveler optimizes the itinerary by minimizing the expected disutility of its travel time (Huang and Gao, 2012). While in *robust optimization*, ambiguity is assumed and a traveler errs on the side of caution and optimizes an itinerary based on the worst-case travel time that might occur within the uncertainty set.

Deadline plays an important role in traveler's itinerary planning. The practices and phenomenon usually occur in urban bus networks that a traveler goes to the destination to participate in a party, to attend a conference, to catch a train, or, most often, to go

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to work. Moreover, even if there is no exogenous deadline imposed at the destination, empirical studies indicate that a traveler may set a target time to arrive as well (Lo et al., 2006; Stopher et al., 2016). Despite its importance, the deadline is rarely considered explicitly in the literature about IPPs under uncertain travel times. Häme and Hakula (2013) adopted a Markov decision process to identify the optimal routing policy, towards maximizing the on-time arrival probability. Due to the inherent complexity of the considered stochastic IPP with deadline, they proposed an approximation algorithm to achieve the computational efficiency. Zhang and Tang (2017) proposed a utility-based decision criterion to hedge against the lateness risk and developed an exact algorithm to solve a stochastic IPP with deadline.

In this paper, we study the *robust itinerary planning problem with deadline* (RIPPD). The RIPPD is described as follows. Given, in a bus network, an origin, a departure time, a destination, and an arrival deadline, we attempt to help the traveler find the most "robust" itinerary that arrives on time as much as possible. Specifically, we characterize the uncertain arc travel times via an adjustable uncertainty set. Our robust model maximizes the size of the uncertainty set while guaranteeing that the corresponding worst-case arrival time of itinerary would not exceed the deadline. We will first focus on the problem without deadline, i.e., the *robust itinerary planning problem* (RIPP), which determines the itinerary with the minimum worst-case travel time for a given uncertainty set. The model and algorithms for the RIPP can be further extended to solve the RIPPD. Inspired by the *budgeted uncertainty set* (BUS) proposed by Bertsimas and Sim (2003, 2004), we propose a *segmented budgeted uncertainty set* (S-BUS) that exploits the characteristics of bus lines. Under the mild assumption that a traveler takes each bus line at most once, the S-BUS enables us to optimally solve the RIPP over a modified network in polynomial time. We focus solely on the "on-time arrival" aspect of the problem and leave for further research the incorporation of other cost components such as bus fare and number of transfers. Noting that the stochastic programming approach that maximizes the on-time arrival probability is generally NP-hard (Xiao et al., 2012), we contribute to the literature by proposing a novel robust optimization approach, which is shown to be computationally efficient and managerially sound.

1.1. Previous related works

The IPPs in the absence of uncertainty have been studied extensively. Dial (1967) was among the first to investigate the IPP in headway-based bus networks, where the transfer waiting time is assumed to be a half the headway of the to-be-transferred bus line. The problem is modeled as a variant of the shortest path problem and solved through a label-setting algorithm (Dijkstra, 1959). Based on Dial's seminal work, extensions were studied in subsequent decades to address several real-world concerns. These extensions, among others, include considering schedule-based public transit services or multi-modal services (Tong and Richardson, 1984; Horn, 2003; Pyrga et al., 2008; Zografos and Androutsopoulos, 2008; De Jonge and Teunter, 2013; Verbas and Mahmassani, 2015), extending to multiple criteria, such as the number of transfers, the travel time, and the monetary cost (Tan et al., 2007; Androutsopoulos and Zografos, 2009; Delling et al., 2014; Yang et al., 2016), and determining a set of *k*-ordered optimal itineraries (Xu et al., 2012; Canca et al., 2013). In addition, the IPPs are important sub-problems for solving transit assignment problems (Hamdouch et al., 2011; Schmöcker et al., 2011; Codina and Rosell, 2017) and transit network design problems (Farahani et al., 2013; Amirgholy et al., 2017). However, most related literature assume deterministic travel times and ignore the uncertainty nature.

Stochastic programming approaches have been employed to address the uncertainty in IPPs. The objective of minimizing the expected total travel time has been studied in stochastic bus networks (Chriqui and Robillard, 1975; Spiess and Florian, 1989; Li et al., 2015b; Verbas and Mahmassani, 2015; Chen and Nie, 2015; Yang et al., 2016). In these studies, optimal policies are identified for travelers who are facing with stochastic transfer waiting times. However, the work that explicitly consider the deadline is rare in the literature about IPP with deadline under uncertain travel times. Häme and Hakula (2013) studied an IPP with deadline in a dynamic stochastic transit network, which is modeled using the Markov decision process. Since the exact method is computational inefficient, they proposed an approximation method to determine the routing policy that maximizes the on-time arrival probability. Zhang and Tang (2017) proposed a utility-based decision criterion to measure the risk of lateness, based on which they developed an exact algorithm to determine the optimal a priori itinerary to meet the deadline as much as possible. The shortest path problems with deadlines, which are related to the IPPs with deadlines, have also been studied in the literature. In these problems, typical objectives include maximizing the on-time arrival probability (Frank, 1969; Nie and Wu, 2009; Xiao et al., 2012; Yang and Zhou, 2017) and minimizing the expected tardiness for the deadline (Verweij et al., 2003; Chen and Zhou, 2010; Cheng et al., 2016), but they generally result in an NP-hard problem (Xiao et al., 2012) and a #P-hard problem (Hanasusanto et al., 2015), respectively. Although these methods might be useful in small networks, it is not easily scalable to solve large-scale problems.

Robust optimization, due to its computational tractability, has become a popular approach of addressing uncertainty in real-world optimization problems (Kouvelis and Yu, 1997; Bertsimas and Sim, 2004; Ben-Tal et al., 2009; Bertsimas and Brown, 2011; Gorissen et al., 2015). Under the assumption of ambiguity, different types of selection criteria and algorithms have been proposed in the literature. The min-max regret criterion was first proposed by Kouvelis and Yu (1997). It was also termed the "robust deviation criterion" or the "relative robustness criterion" in the context of robust shortest path problems (Montemanni and Gambardella, 2004). Karaşan et al. (2001) formulated the robust shortest path problem with this criterion as a mixed integer programming. Subsequently, several methods were proposed to solve this problem (Montemanni and Gambardella, 2004, 2005; Catanzaro et al., 2011). However, the problem was proved to be NP-hard (Zieliński, 2004). Another robustness criterion, termed the "*bw*-robustness criterion", was studied by Gabrel et al. (2013) in the context of shortest path problems, but it still leads to an NP-hard problem. Shahabi et al. (2013) considered the case where only mean and covariance of arc travel times are known. They adopted the mean plus standard deviation criterion to define "robust" and proposed an out approximation algorithm to solve the robust shortest path problem. An exception to make a robust counterpart of the shortest path problem intractable is the "worst-case criterion". This criterion was first proposed by