Exact and heuristic algorithms for finding envy-free allocations in food rescue pickup and delivery logistics

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ABSTRACT

Food rescue organizations collect and re-distribute surplus perishable food for hunger relief. We propose novel approaches to address this humanitarian logistics challenge and find envy-free allocations of the rescued food together with least travel cost routes. We show that this food rescue and delivery problem is NP-hard and we present a cutting-plane algorithm based on Benders' decomposition for its exact solution. We introduce a novel heuristic algorithm that combines greedy and local search. We test our approaches using real data from food rescue organizations. Our results show that the proposed algorithms are able to efficiently provide envy-free and cost-effective solutions.

1. Introduction

Hunger is one of the main humanitarian challenges faced by emerging and developed countries. One of the externalities of our consumer society is that food is often wasted. For instance, to keep up with competitors, grocery stores continually store fresh food and have a tight product turnover which typically leads to extensive waste. The same holds for catering events which often prefer to overestimate than underestimate attendance. In an attempt to reduce this societal imbalance, charities and not-for-profit organisations have promoted food rescue initiatives and engineered logistical solutions to re-distribute rescued food to welfare agencies (Foodbank US, Australia; Second Harvest Canada; Food Rescue Australia; OzHarvest Australia; Secondbite Australia; Foodbank Singapore; City Harvest US; Food Shuttle US; Feeding India, India; Leket Israel, etc.) (OzHarvest, 2017; SecondBite, 2017; FoodBank, 2017; SecondHarvest, 2017; Leket Israel, 2017; Feeding India, 2017). The past several years have seen a burgeoning in food rescue operations with recovering tons of excess food and distributing it to the people in need. Foodbank Australia delivers 65 million meals per year whereas OzHarvest Australia re-distributes 5 million kg of food per year. In comparison, Food bank Central and Eastern North Carolina delivers 30 million kg of food per year, Feeding America claims to feed 46 million people per year and Second Harvest Canada delivers 11 million meals per year.

It is frequent that the total request of welfare agencies exceeds the available food supply. Hence, it is necessary to fairly allocate the rescued food among welfare agencies. In addition, food rescue organizations typically focus on collecting perishable food which must be re-distributed on the same day, without any storage possible. Due to the perishability of food products collected, these products cannot always be stored in warehouses, and same-day deliveries may be required. This is particularly relevant for the delivery of ready-to-serve meals.

Food rescue programs have become increasingly popular and so has the scale of their operations, thus motivating the need for

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customized quantitative decision-making frameworks. From a logistical standpoint, the food rescue and delivery problem involves two types of decisions: food allocation and distribution. The former is concerned with the problem of finding fair or equitable allocations of the rescued food among welfare agencies. In turn, the latter is focused on identifying cost-effective vehicle routes to pick-up and deliver the rescued food. While allocation and routing problems have received a significant attention in the literature, the vast majority of the proposed approaches focus on cost-driven solutions which are oblivious to fairness considerations. In this paper, we propose novel solution methods for the fair allocation and re-distribution of rescued food.

Our approach is designed to promote envy-free allocations which are fair with regards to welfare agencies' requests. An allocation is said to be envy-free if all the welfare agencies receive the same amount of food or if the welfare agencies which receive less than the maximum amount delivered are satisfied. Although envy-freeness is a desirable property of fair division solution methods, such equitable allocations may not exist, especially in the context of resource-constrained logistical operations. We highlight this challenge on a toy problem and show that this food rescue and delivery problem is NP-hard. We present a mixed-integer linear program (MILP) representation of the problem which combines fair allocation and vehicle routing decisions. We propose a cutting-plane algorithm based on Benders' decomposition for its exact solution. We then introduce a heuristic algorithm for finding (near) envy-free and cost-effective solutions for the food rescue and delivery problem. Our approach is evaluated through numerical experiments designed to be representative of realistic food rescue programs.

This work builds on prior research by Nair et al. (2016, 2017a, 2017b), especially the latter. In Nair et al. (2016, 2017a) the effort is focused on minimizing routing cost for periodic pick-up and delivery vehicle routing problems *i.e.*, there is no food allocation component within the optimization problems solved therein. In Nair et al. (2017b), the authors solve a food rescue and delivery problem similar to that of the present paper with different fairness criteria, *i.e.*, egalitarian and maximum deviation *versus* envy-freeness; and propose a meta-heuristic based solution approach. In contrast, in the present paper we combine relaxations and valid inequalities in a novel cutting-plane algorithm for the exact solution of food rescue and delivery problems and introduce a new heuristic algorithm based on local search.

The paper is organized as follows. Section 2 reviews the state-of-the-art on humanitarian logistics as well as allocation and routing solution approaches. Section 3 formally introduces the food rescue and delivery problem, its mathematical programming representation and a cutting-plane algorithm for its exact solution. Section 4 presents a novel heuristic approach for the food rescue and delivery problem. Section 5 demonstrates the performance of the proposed heuristic algorithm through numerical experiments and Section 6 summarizes the findings of this paper and outlines possible extensions.

2. Literature review

We first review the literature on vehicle routing with pickup and delivery operations which is closely related to the routing component of the food rescue and delivery problem. We then present the main relevant efforts in humanitarian logistics before focusing on fairness-driven allocation and routing approaches.

2.1. Vehicle routing for pickup and delivery operations

The routing component of the food rescue and delivery problem can be represented as a single-commodity pick-up and delivery vehicle routing problem. We hereby briefly review the literature on pick-up and delivery routing problems and position our paper in this field.

In the context of the traveling salesman problem (TSP) Hernández-Pérez and Salazar-González (2004a) introduced the single-commodity, unpaired pickup and delivery problem (1-PDTSP) which has many real-world applications such as the transportation of milk, sand, gas, eggs or vaccines. They proposed integer-linear programming formulations and a branch-and-cut algorithm able to solve instances with up to 40 customers. They also proposed a heuristic approach based on a greedy algorithm improved with a *k*-optimality criterion for the same problem (Hernández-Pérez and Salazar-González, 2004b) and latter extended this work by introducing new inequalities (Hernández-Pérez and Salazar-González, 2007). Other approaches for the unpaired PDTSP are based on different heuristic algorithms including a hybrid algorithm combining greedy randomized adaptive search procedure (GRASP) and variable neighborhood descent (VND) (Hernández-Pérez et al., 2009), a genetic algorithm approach (Zhao et al., 2009), an exact permutation algorithm (Martinovic et al., 2009) and a variable neighborhood search (VNS) algorithm (Mladenović et al., 2012).

In the case of multiple vehicles, Chen et al. (2014) introduced the multi-commodity unpaired pickup and delivery vehicle routing problem (PDVRP) with split loads and presented a mathematical formulation and a VNS algorithm to find near optimal solution for instances with up to 10 customers and 6 commodities. Recently, Xu et al. (2017) presented a multi-visit unpaired PDVRP, which is an extension of the model presented by Chen et al. (2014). They proposed a tabu search solution approach and the heuristic evaluation study reports an average and maximum gap between the best solution and its lower bound of 33.04% and 83.1% for instances with 5 products and a number of customers ranging from 5 to 10. Azadian et al. (2017) presented an unpaired PDVRP with time-dependent assignment cost and implements an efficient solution method based on a decomposition approach for an air cargo transportation problem. The solution method is evaluated by comparing the best solution with the optimal solution (for instances of the problem cases with 7 customers and 1 airport) and lower bound values (instances of the problem cases with 15 customers and 2 airports). For large instances, the average gap reported is approximately 11% with a worst-case scenario of 22.7% gap against the lower bound. However, these models are significantly different from the model presented in the current study where we jointly model allocation and routing decisions under limited supply conditions. Recently, Nair et al. (2016, 2017a) presented a periodic unpaired PDVRP and proposed a tabu search based heuristic solution approach. These studies focus mainly on the scheduling aspect of the logistics