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A note on lead-time paradoxes and a tale of competing prescriptions

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1. Introduction

Global competition and long, complex supply lines place unyielding pressure on firms to cope with replenishment lead-time uncertainty and to reduce inventory without hurting product availability. If asked what they expect from either less variable or shorter lead-times, most supply chain professionals would say lower inventories. They would be astonished to discover that these actions may paradoxically increase inventory and total cost. All else being equal, a supplier with shorter lead times may generate a higher inventory-system cost than a supplier with longer lead times, while a supplier with less variable lead times may require more safety stock than a supplier with more variable lead times. Most importantly, these possibilities emerge when commonly-used statistical distributions characterize random lead times, including the normal, gamma, lognormal, Weibull, and uniform distributions.

Although such counterintuitive possibilities exist, little is known about the origins of lead-time paradoxes and the real risks of managing lead time incorrectly. Indeed, seminal studies disagree on the risks and offer competing prescriptions for managing them. This research note critically reviews these studies to develop deeper insights into the sources, risks, and practical significance of lead-time paradoxes. The next section of the paper reveals the origins of the paradoxes. Subsequent sections critically review the risks of pulling the wrong lead-time levers and present an in-depth assessment of the conditions that produce competing management prescriptions. The paper closes with the conclusions and managerial implications. Additionally, a Glossary describes the notation, while Appendix A summarizes the underlying methods and techniques found in the logistics, transportation, and decisions sciences literature.

2. Origins of lead-time paradoxes

Lead-time paradoxes have roots in decision theory situations in finance and economics, where the evaluation of random prospects uses stochastic dominance and mean-variance (M-V) methods. Stochastic dominance determines the superiority of one random prospect over another. The M-V approach orders prospects by holding the means constant while experimenting with the variances, or holding the variances constant while experimenting with the means (Bagchi et al. 2007, p. 18). If the expected payoff of two random prospects is the same, for example, the prospect with the lower variance is preferred. Other the other hand, if the variance of the two prospects is the same, the prospect with a higher payoff is preferred. Although this example is intuitive, research on the definition of "riskier" prospects confirms that it is possible for the prospect with the same or smaller mean but a higher variance to have the same or higher expected utility (Levy, pp. 562, 567). In other words, the counterintuitive possibility exists for a prospect offering the same expected payoff with less variability to have less value.

Song (1994) revealed the existence of lead-time paradoxes in a seminal study that used stochastic dominance and M-V techniques to investigate the effects of lead-times and their variabilities on optimal inventory policies and system performance. The investigation considers a special case of a continuous review system for one item, with reorder point R, order quantity Q, and no fixed order cost which enables an optimal base-stock policy corresponding to $Q = 1$. In a sequel to this study, Song et al. (2010) extended the findings

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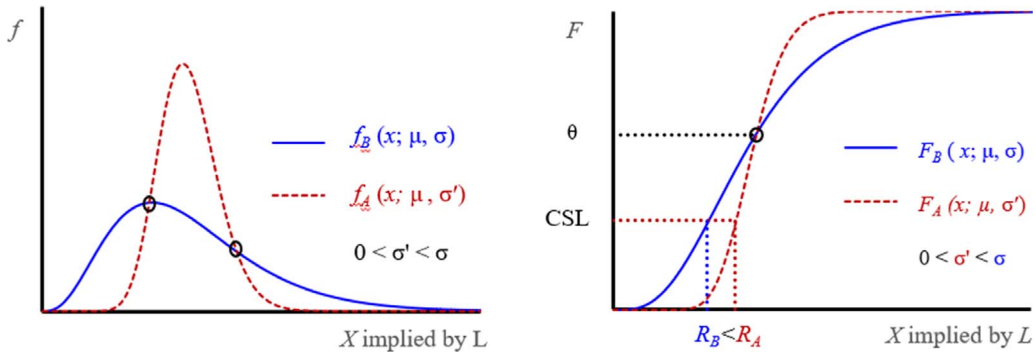

Fig. 1. Stochastically less variable lead time (L) or lead-time demand (X) of supplier A in relation to supplier B.

to a continuous review system in which holding, backordering, and fixed ordering costs constitute total cost (TC).

The inventory policies respond to independent demand (D), which occurs in epochs that follow a Poisson process with a random batch size in each epoch and to an exogenous random lead-time (L) process that is independent of demands and orders. The setting includes two random lead-time prospects: (1) supplier B has the same mean lead time as supplier A, but a stochastically less variable lead time; (2) supplier B has the same lead time variance as supplier A, but a stochastically shorter lead time. The evaluation considers each prospect under identical conditions.

2.1. Effects of less variability on R and safety stock

Song (1994) defines a "stochastically less variable" L by the number and direction of intersections between the densities (f) of two random variables with the same functional form and mean (μ), but different standard deviations (σ). The lead-times of supplier A are stochastically less variable than the lead times of supplier B when the lead-time density function of supplier A intersects the density function of supplier B exactly twice, first from below and then from above as illustrated on the left side of Fig. 1 for f_A and f_B. This pattern, or ordering of the two densities, implies that the distribution function F_A will intersect the distribution function F_B exactly once from below as shown on the right side of Fig. 1. Additionally, a less variable L by this definition results in a less variable lead-time demand distribution (X) for commonly-used unimodal distributions of L, including gamma, Weibull, normal, truncated-normal, and lognormal distributions (Song, 1994, p. 609).

When F represents the distribution of X, it provides the in-stock probability $Pr(X \leq x)$ that defines the cycle service level (CSL). If the CSL is below the threshold (θ) that is defined by the point where F_A intersects F_B from below, the less variable lead-time demands of supplier A result in a higher R and safety stock ($R - \mu_x$) than the more variable lead-time demands of supplier B.

Several insights emerge from the less variable L construct. First, the ordering of the unimodal densities in Fig. 1 illustrates the iconic pattern of second order stochastic dominance (Vose, 2008, p. 304). Second, the inverse construct, or a "stochastically more variable" L, aligns with the definition of a 'riskier" prospect, which encompasses more variance and more tail weights (Levy, 1992, p. 562; Rothschild and Stiglitz, 1970). Third, and most importantly, the M-V approach creates this pattern in the commonly-used unimodal densities of L. A decrease in σ without changing μ decreases the scale of the L, which compresses f_A from both sides toward μ when plotted on a fixed-interval x-axis. As a consequence, f_A appears taller and thinner than f_B, and has less weight in the tails. The decrease in scale also makes F_A appear to rotate counter clockwise into a more vertical position that intersects F_B from below. The M-V approach is not a strict condition for θ to emerge, because a very small decrease in μ coupled with a very large decrease σ may produce the requisite intersection. Finally, the two-supplier construct is equivalent to a single-suppler construct when f_B is the lead-time density before an independent change in either σ or μ and f_A is the density after the change.

2.2. Effect of shorter lead time on R and safety stock

Given the same σ, but a different μ, supplier A has a stochastically shorter lead time than supplier B, if the lead-times of supplier A have a larger probability of being less than or equal to the lead times of supplier B. In other words, $F_A \geq F_B$ for each possible l as illustrated on the right side of Fig. 2. This ordering of F exhibits first-order stochastic dominance (Vose, 2008, p. 304).

Most importantly, because $\mu_X = \mu_L \cdot \mu_D$, the distribution of X inherits this ordering, so $F_A \geq F_B$ for all x (Song 1994, p. 608; Song et al. 2010, p. 74). A decrease in μ_L results in a decrease in μ_X, which shifts the location of F_A leftward in relation to F_B when plotted on the same fixed-interval x-axis as illustrated in Fig. 2. A shorter L implies that $R_A \leq R_B$ for all CSL, which almost always results in a lower R and safety stock for supplier A than for supplier B.

3. Risks of pulling the wrong lead-time lever

The risks of pulling the wrong lead-time lever involve the prospect that TC may increase after decreasing either lead-time uncertainty (σ) or lead-time (μ). The seminal literature provides profoundly conflicting views of these risks. On one hand, the original