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Capacitated path-aggregation constraint model for arc disruption in networks

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ABSTRACT

Path-aggregation constraint (PAC) models can be used to represent flows across networks with specific origin-destination demand pairs without requiring explicit enumeration of all potential paths, greatly reducing the computational burden needed analyze a network's performance in the event of arc disruptions. This paper presents new PAC models for examining arc disruptions on networks, providing the first known models to extend such PACs to capacitated arcs. Extensive computational testing demonstrates that these novel arc-capacitated models do not significantly increase computational burden beyond that required by arc-uncapacitated models, while avoiding the suboptimal performance achieved when uncapacitated models are applied to arc-capacitated networks.

1. Introduction

Connectivity is one of the most commonly used metrics for assessing the system performance of networks. For analyses of system performance in the event of disruption to network components (i.e., disruption to arcs and/or nodes), connectivity-based metrics focus on identifying the existence (or not) of a path in the network between node pairs following the disruption event. However, in some applications, the use of a connectivity metric can mask severe degradation to network performance following a disruption. For example, the failure of a bridge (e.g., I-35 across the Mississippi river in 2007, I-40 across the Arkansas River in 2002) might not cause a loss in connectivity provided that alternative routes exist between all origins and destinations, but such a loss can generate significant travel delays and economic losses as people and goods are forced to utilize these less-efficient alternative routes (National Transportation Safety Board, 2002; Xie and Levinson, 2011). Similarly, failures on a computer network, such as those experienced at the University of California in 1986, can lead to significant reductions in connection speed even though connectivity is maintained to all computer nodes (Jacobson, 1988).

More generally, use of connectivity metrics for system performance is problematic for disruption analyses of networks that feature a capacity on the maximum-allowable flow across arcs. For transportation systems serving demands between source-destination node pairs (denoted *s-t* pairs), such an analysis of the system's performance should capture not only the existence of connectivity between *s-t* pairs after a disruption, but also flow-based metrics, such as the lost flows of *s-t* pair demands that cannot be satisfied (or that can only be partially satisfied), or the delays encountered due to disruption across *s-t* pairs for which connectivity remains. A challenge to incorporating arc capacities into such analyses is that the flow across multi-arc paths must now be tracked, increasing the computational difficulty of analysis.

One means to potentially mitigate the impact of disruptions to the arcs and/or nodes of a network is by employing a set of

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protective resources to network components that render these arcs and/or nodes immune to disruption. Clearly, protecting all network components would be desirable. However, this approach is typically not practical, due to constraints on the available personnel and material needed to perform the necessary protective activities. The network manager responsible for ensuring the access of flows between origin and destination pairs needs to know which sets of arcs would, in the event of their disruption, lead to the maximum loss of satisfied demands. This information can help inform investment decision on the part of the manager with respect to arc protection. From another perspective, this information would also be useful to a network attacker, interested in identifying the optimal subset of arcs to disable in order to disrupt these flows between origins and destinations.

This paper introduces mixed-integer linear programming (MILP) models to identify a subset of arcs for protection against disruption in arc-capacitated networks, assessing alternative protection investments against the worst-case scenario, defined here as the scenario generating the maximum potential disrupted *s-t* flow. Accordingly, Section 2 presents a review of research assessing network performance in the event of disruptions across a variety of objectives and application areas. Section 3 presents our mathematical formulations, in which we extend the computationally-efficient PAC (path aggregation constraint) MILP approach of Matisziw and Murray (2009a) so that it captures lost system flows due to insufficient capacity. The primary contribution of this paper is its extension to methodology: by use of a PAC approach, our models are able to solve much larger problem instances than the traditional path-enumerating techniques, and this paper is the first to present a PAC model for critical arc identification that takes into consideration lost flows due to arc capacities. Section 3 further provides more-efficient alternative PAC MILP formulations incorporating novel valid inequalities for both uncapacitated and arc-capacitated flow networks. Section 4 presents numerical experiments demonstrating the computational performance of the proposed models on a set of publicly-available network flow examples. Section 5 then presents conclusions and suggestions for further research.

2. Literature review

In the literature, many authors (such as Matisziw et al., 2010; Matisziw and Murray, 2009a; Murray et al., 2007; Myung and Kim, 2004), have developed models for allocating a limited set of protective resources in an aim to fortify the network against disruption by identifying the p component worst-case failures. Here, the p component worst-case (also referred to as the p most-critical arcs (or nodes)) refers to the p arcs (or nodes) which, when disrupted, cause the greatest degradation in network performance.

Travel distance (Cappanera and Scaparra, 2011; Israeli and Wood, 2002), travel time (Jin et al., 2015; Ukkusuri and Yushimito, 2009), maximum capacity flow (Ford and Fulkerson, 1962; Wood, 1993) and network connectivity (Arulselvan et al., 2009; Matisziw and Murray, 2009b) are some of the performance metrics that have been utilized in the literature to examine how a network is affected by such p component disruptions. As discussed in Section 1, connectivity-based system performance metrics can provide limited information for networks with capacity constraints on arcs. With respect to the specific problem examined in this paper, the most relevant performance metric is the *lost system flow* in the context of p component failures. Were we to remove p arcs from such a network, even if connectivity remains between all nodes, lost flow could occur due to the capacities on the remaining arcs. Numerous authors have examined the impacts of arc capacity on such networks, and the impacts of disruptions to their arcs, in applications such as transportation and supply chain networks (Chen et al., 2002; Harris and Ross, 1955; Morlok and Chang, 2004; Scaparra and Church, 2010), communication networks (Aggarwal et al., 1982; Stoer and Dahl, 1993; Yu and Yang, 2011), and social networks (Schneider, 2013). Bell et al. (2017) studied network vulnerability on undirected capacitated networks with no origin-destination demand pairs using weighted spectral analysis, identifying arc sets that are similar in some respects to the concept of a min-capacity cut in max flow problems. Identifying the optimal allocation of protective resources on such networks' arcs requires that our measure of impact on network performance for a given disruption scenario captures these arc capacity effects.

Existing MILP formulations for the lost flow performance metric require enumeration of all *s-t* pair paths for the identification of the p most critical components, whether arcs (Myung and Kim, 2004), nodes (Murray et al., 2007), or a combination thereof (Matisziw et al., 2010; Scaparra et al., 2015; Starita and Scaparra, 2016). Such enumeration within the MILP formulation allows one to assess survivable paths after p component interdictions to the network. However, the path enumerating process can be computationally expensive and models depending on this process can be inadequate for application to large networks due to intractability. Matisziw and Murray (2009a) proposed an alternative PAC MILP system flow formulation for uncapacitated networks, eliminating the requirement for such path enumeration. Here, the PAC formulation captures all available paths maintaining connectivity after disruption to p arcs. Unlike the classical multi-commodity flow model that balances flows between a set of supply nodes and a set of demand nodes (Dantzig, 1963), this PAC structure can be used to account for demands between specific *s-t* pairs. However, the usage of connectivity-based approach in Matisziw and Murray (2009a) could lead to a sub-optimal allocation of the protective resources in a network with arc capacities. Therefore, this paper aims to identify the p most-critical arcs in terms of disrupted flow on arc-capacitated networks, without requiring enumeration of paths (which is computationally intractable for large problems), but instead capturing path information using the efficient PAC approach of Matisziw and Murray (2009a) applied to capacitated arcs, an extension that these authors claimed could not be incorporated into their PAC model.

3. Mathematical formulations

The remainder of this paper will utilize the following notation for the PAC MILP flow formulations. Define directed network $G = (N,A)$ comprised of n nodes and m arcs. Denote the set of nodes by N, the set of arcs by A, and the subsets of demand origin nodes and demand destination nodes by S and D, respectively. It is further assumed that it is possible for a single node to be a member of both subset S and subset D. Note that while the models examined in this paper address arc failures on directed networks, these models