Optics and Lasers in Engineering 70 (2015) 18–25

Contents lists available at ScienceDirect

Optics and Lasers in Engineering

journal homepage: www.elsevier.com/locate/optlaseng

Absolute phase recovery of three fringe patterns with selected spatial frequencies

Yi Ding [a], Jiangtao Xi [b,*], Yanguang Yu [b], Fuqin Deng [c]

[a] Shenzhen Institution of Advanced Technology, Chinese Academy of Science, 1068 Xueyuan Avenue, Shenzhen 518055, China
[b] School of Electrical Computer and Telecommunications Engineering, University of Wollongong, Wollongong, NSW 2522, Australia
[c] Shenzhen Graduate School, Harbin Institution of Technology, Shenzhen 518055, China

ARTICLE INFO

Article history:
Received 25 August 2014
Received in revised form
26 November 2014
Accepted 28 December 2014
Available online 6 March 2015

Keywords:
Fringe analysis
Phase shift
Three-dimensional sensing
Phase unwrapping
Surface measurements

ABSTRACT

A new temporal approach is presented for the recovery of the absolute phase maps from their wrapped versions based on the use of fringe patterns of three different spatial frequencies. In contrast to the two-frequency method recently published, the method proposed is characterized by better anti-error capability as measured by phase error tolerance bound. A general rule for the selection of the three frequencies is presented, and its relationship to the phase error tolerance bound is derived. Theoretical analysis and experimental results are also presented to validate the effectiveness of the proposed three frequency technique.

1. Introduction

Fringe projection profilometry (FPP) is one of the most promising technologies for non-contact 3D shape measurement. A challenging task associated with existing phase measurement technique in FPP is phase unwrapping operation, which aims to recover the absolute phase maps from the wrapped phase maps. Existing phase unwrapping methods include spatial [1], temporal [2,3], and period coding [4]. However, recovery of absolute phase maps is still a challenging task when the wrapped phase maps contain noise, sharp changes or discontinuities [5].

To achieve reliable and accurate phase unwrapping for FPP, a variety of temporal phase unwrapping approaches have been proposed following work of Huntley and Saldner [2]. The general idea behind this temporal method is that multiple fringe patterns are projected onto the object, yielding a sequence of wrapped phase maps as a function of time t. These phase maps can be considered as a 3D phase map $\phi(m,n,t)$, denoting the wrapped phase value at pixel (m,n) at the tth phase map ($t=0, 1, 2, \ldots, s$). Phase unwrapping can be carried out along any path in the 3D space in order to avoid noise or boundaries and thus achieving correct recovery of the absolute phase map. While the method proposed in [2] is demonstrated to be effective for accurate phase unwrapping, it also suffers from the drawback of requiring many intermediate phase patterns (e.g., 7 sets of fringe patterns were employed in [2]), which is obviously not suitable for fast or real-time measurement. In order to increase the efficiency, Zhao et al. [3] propose to use two image patterns, one of which has a very low spatial frequency in contrast to the other. In particular, the low spatial frequency pattern only has a single fringe. Such a pattern has its absolute phase value falling within the range $(-\pi,\pi)$, and hence can be used as a reference to calculate the fringe number of the other fringe pattern, thus yielding its absolute phase map. Li et al. [5,6] also employ the phase map of single fringe pattern as reference to unwrap high spatial frequency fringe patterns, and it is shown that the spatial frequency of the pattern to be unwrapped is determined by the level of noise. Following the same method in [5], Liu et al. [7] project a single fringe pattern and a high frequency pattern in one shot to accelerate the speed of 3D measurement. This method works well in principle, but the gap between two spatial frequencies should be restricted within a range based on the noise level or steps in the low frequency phase maps. As the accuracy performance of FPP requires the use of high frequency fringe patterns, these methods may not work well when the phase maps are noisy or discontinuous. Consequently, multiple intermediate image patterns are still required in order to reduce the frequency gaps among adjacent patterns. Saldner and Huntley [8,9] study the multiple intermediate image patterns, showing that to unwrap a phase map of frequency f, $\log_2 f+1$ sets of fringe patterns are required. A similar result is also reached by Zhang [10,11], indicating

* Corresponding author.
E-mail address: jiangtao@uow.edu.au (J. Xi).

http://dx.doi.org/10.1016/j.optlaseng.2014.12.024
0143-8166/

that the spatial frequency can be increased by a factor of 2 between two adjacent patterns. Taking a typical FPP arrangement as an example where the image pattern has 16 fringes, 5 image patterns are still required with this approach.

In order to recover the absolute phase maps of high spatial frequency fringes with less number of fringe patterns, we have developed a temporal phase unwrapping technique based on the use of two fringe images with two selected frequencies [12]. When the two normalized spatial frequencies f_1 and f_2 are coprime, there exists a one-to-one map from $[f_2\phi_1(x)-f_1\phi_2(x)]/2\pi$ to their fringe orders, where $\phi_1(x)$, $\phi_2(x)$ are the wrapped phase maps. We also obtain the minimal value gap of $[f_2\phi_1(x)-f_1\phi_2(x)]/2\pi$ when f_1 and f_2 are coprime. However, the performance of the proposed method in [12] is limited by phase error tolerance bound, $\pi/(f_1+f_2)$ [13]. If the phase error of wrapped phase maps is larger than the phase error bound, errors may occur in the recovery of the absolute phase maps. As demonstrated by the experiments in [13], phase errors in many practical situations are significant and can easily exceed the bound, leading to the failure in recovering the absolute phase map. Therefore, it is desirable to develop new approaches with the aim to increase phase error tolerance bound. To this end, we propose a method based on the projection of three fringe patterns with selected frequencies. The idea is that with the use of three spatial fringe patterns, the minimal value gap on the values of $[f_2\phi_1(x)-f_1\phi_2(x)]/2\pi$ can be increased to higher than one, resulting in a higher phase error tolerance bound.

Zhong et al. [14] also constructed a look-up table to unwrap the absolute phase maps for multiple-spatial-frequency fringes. This look-up table denotes the corresponding relationship from a pair of fringe orders at two spatial frequencies (f_1, f_2) to $[f_2\phi_1(x)-f_1\phi_2(x)]/2\pi$. When the spatial frequencies f_1 and f_2 are large values, one value in $[f_2\phi_1(x)-f_1\phi_2(x)]/2\pi$ may correspond to two or more pairs of fringe orders, thus the fringe orders cannot be determined uniquely. To make sure the values of $[f_2\phi_1(x)-f_1\phi_2(x)]/2\pi$ unique, Zhong, et al. [15] proposed to use relatively irrational spatial frequencies for the wrapped phase maps, that is, $f_1=3, f_2=5, f_3=3\sqrt{2}/2$ (not the normalized spatial frequencies). To apply the relatively irrational frequencies, Zhong [16] proposed to generate the two relatively irrational spatial frequencies fringes by changing the projection angle of the grating. However, the spatial frequency selection in [14] does not guarantee the one-to-one map from $[f_2\phi_1(x)-f_1\phi_2(x)]/2\pi$ to a pair of fringe orders, thus the dynamic measurement range is smaller than the section of pattern image [15]. Furthermore, the minimal value gap of $[f_2\phi_1(x)-f_1\phi_2(x)]/2\pi$ of two irrational frequencies is always smaller than the two rational frequencies [14,15,16], which may yield mistakes in determining fringe order pairs. Our proposed method could guarantee the one-to-one map and increase the minimal value gap of $[f_j\phi_i(x)-f_i\phi_j(x)]/2\pi$ significantly to enhance the reliability of absolute phase maps.

This paper is organized as follows. In Section 2 we present the technique to recover the absolute phase maps with three selected frequency fringe patterns. In Section 3, we give the principle to increase the smallest value gap by selecting frequencies. In Section 4, experiments are presented to validate the effectiveness of three frequency technique and the principle to increase the value gap. Section 5 concludes the whole paper.

2. Absolute phase maps recovery with three frequency fringe patterns

2.1. Three frequency technique

Let us consider a FPP system, with which three image patterns are projected onto the object surface respectively. The image patterns are characterized by fringe structure where the light intensity is constant in y-axis and varies sinusoidally in x-axis. The normalized spatial frequencies of the three patterns are f_1, f_2 and f_3, referring to the total number of fringes on the respective patterns. Let us use $\Phi_i(x)$ $(i=1,2,3)$ and $\phi_i(x)$ $(i=1,2,3)$ to denote respectively the absolute phase maps and the corresponding wrapped phase maps of the fringe patterns. Taking the central vertical line of images as the reference, the value of the wrapped phase map is limited by $-\pi \le \phi_i(x) \le \pi$ $(i=1,2,3)$, and the value of the absolute phase maps should fall into the following:

$$-f_1\pi \le \Phi_1(x) \le f_1\pi,\ -f_2\pi \le \Phi_2(x) \le f_2\pi,\ -f_3\pi < \Phi_3(x) < f_3\pi \tag{1}$$

Hence the absolute and wrapped phase maps are related by the following:

$$\Phi_i(x) = 2\pi m_i(x) + \phi_i(x) \tag{2}$$

Where $m_i(x)(i=1,2,3)$ are referred to as fringe numbers or indices. They are integers and $-\lfloor f_i/2 \rfloor < m_i(x) < \lfloor f_i/2 \rfloor$ $(i=1,2,3)$. Obviously, the absolute phases can be recovered if $m_i(x)(i=1,2,3)$ are determined. In order to achieve this, we employ the following relationships [10]:

$$f_2\Phi_1(x) = f_1\Phi_2(x),\ f_3\Phi_1(x) = f_1\Phi_3(x) \tag{3}$$

Combining Eqs. (2) and (3), we have:

$$\frac{f_2\phi_1(x)-f_1\phi_2(x)}{2\pi} = m_2(x)f_1 - m_1(x)f_2,\ \frac{f_3\phi_1(x)-f_1\phi_3(x)}{2\pi} = m_3(x)f_1 - m_1(x)f_3 \tag{4}$$

Similar to the method employed in [12,13], an intermediate variable $\Phi_0(x)$ is introduced, which increases monotonically from $-\pi$ to π with respect to x and defined as follows:

$$\Phi_0(x) = \frac{\Phi_1(x)}{f_1} = \frac{\Phi_2(x)}{f_2} = \frac{\Phi_3(x)}{f_3} \tag{5}$$

Considering $\Phi_0(x) = \Phi_1(x)/f_1$ and taking account of Eq. (1), $m_i(x)$ $(i=1,2,3)$ can be determined by the value of $\Phi_0(x)$ as follows:

$$m_1(x) = \begin{cases} \lfloor f_1/2 \rfloor & [f_1-(f_1 \bmod 2+1)]\pi/f_1 \le \Phi_0(x) < \pi \\ \cdots & \cdots \\ 1 & \pi/f_1 \le \Phi_0(x) < 3\pi/f_1 \\ 0 & -\pi/f_1 < \Phi_0(x) < \pi/f_1 \\ -1 & -3\pi/f_1 \le \Phi_0(x) < -\pi/f_1 \\ \cdots & \cdots \\ -\lfloor f_1/2 \rfloor & -\pi < \Phi_0(x) \le -[f_1-(f_1 \bmod 2+1)]\pi/f_1 \end{cases} \tag{6}$$

$$m_2(x) = \begin{cases} \lfloor f_2/2 \rfloor & [f_2-(f_2 \bmod 2+1)]\pi/f_2 \le \Phi_0(x) < \pi \\ \cdots & \cdots \\ 1 & \pi/f_2 \le \Phi_0(x) < 3\pi/f_2 \\ 0 & -\pi/f_2 < \Phi_0(x) < \pi/f_2 \\ -1 & -3\pi/f_2 < \Phi_0(x) \le -\pi/f_2 \\ \cdots & \cdots \\ -\lfloor f_2/2 \rfloor & -\pi < \Phi_0(x) \le -[f_2-(f_2 \bmod 2+1)]\pi/f_2 \end{cases} \tag{7}$$

$$m_3(x) = \begin{cases} \lfloor f_3/2 \rfloor & [f_3-(f_3 \bmod 2+1)]\pi/f_3 \le \Phi_0(x) < \pi \\ \cdots & \cdots \\ 1 & \pi/f_3 \le \Phi_0(x) < 3\pi/f_3 \\ 0 & -\pi/f_3 < \Phi_0(x) < \pi/f_3 \\ -1 & -3\pi/f_3 < \Phi_0(x) \le -\pi/f_3 \\ \cdots & \cdots \\ -\lfloor f_3/2 \rfloor & -\pi < \Phi_0(x) \le -[f_3-(f_3 \bmod 2+1)]\pi/f_3 \end{cases} \tag{8}$$