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Fast Hermite element method for smoothing and differentiating noisy displacement field in digital image correlation

Jiaqing Zhao*, Yu Song, Xinxin Wu

Institute of Nuclear and New Energy Technology of Tsinghua University, The Key Laboratory of Advanced Reactor Engineering and Safety, Ministry of Education, Beijing 100084, China

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ABSTRACT

In our previous work, an improved Hermite finite element smoothing method (IHFESM) combined with the well-known Tikhonov regularization was proposed to smooth the noisy displacement field directly calculated by digital image correlation (DIC). Even though IHFESM could reconstruct reliable strain field for arbitrary region of interest (ROI), it still suffers from three defects in practice: (i) for the large scale problems, it involves high computational burden to find the optimum regularization parameter within the generalized cross-validation (GCV) function, since the inverse and the trace of two large matrices are frequently evaluated, (ii) the search range is too wide to find the desired optimum regularization parameter quickly, and (iii) for the arbitrary ROI, the complex two dimensional meshing routines are required to locate and mesh the invalid regions into triangular elements. In current work, to overcome above deficiencies, we propose the fast Hermite element method (FHEM). The proposed FHEM avoids the redundant computation in IHFESM by decomposing the resultant two small matrices to speed up the original matrix inverse and trace computation. To narrow down the search range of regularization parameter, the magnitude of the closeness term and roughness penalty term in error function are balanced by a new parameter. The calculation of global regularization matrix is also sped up by using simple formulation without any sophisticated meshing routines. Experiments show that the FHEM is at least 50 times as fast as IHFESM with similar accuracy, and it is recommended to be adopted to smooth and differentiate the noisy displacement field for DIC in practice.

1. Introduction

Digital image correlation (DIC) [1,2] is an effective optical non-contact full-field strain measurement method which is widely used in the field of experimental mechanics. Similar to all the other measurement techniques, improvement of the measurement accuracy is always a hot topic for DIC [3–7]. As we all know, during the imaging process using CCD sensors, there are many sources of noises [8,9] (such as shot noise, thermal noise, readout noise and photon noise). These noises finally cause the unavoidable gray scale noise in the captured images. Theoretical analysis shows that the variance of displacement is proportional to the deviation of the image noise, which usually conforms to the independent Gaussian distributions with zero mean [10]. As the combination of the gradient of the displacement field, the strain field is also contaminated, especially in the case of small deformation. One way to decrease the image noise level is to employ the CCD camera of higher signal–noise ratio [11], however this will increase the hardware cost greatly. More researchers have focused on developing the effective post-processing techniques to filter out the noise in displacement and obtain the smoothed strain fields, and these methods could be classified into two main catalogs [12]: (1) the local smoothing method, such as the pointwise least squares [5,13] and diffuse approximation [14], (2) the global smoothing method, such as the thin-plate spline smooth method [15,16] and the finite element based smoothing methods [3,6,12,14,17].

In our previous work in [12], we put forward a global smoothing method, or exactly the improved Hermite finite element smoothing method (IHFESM), which used the C^2 continuous (second order continuous) Hermite finite elements to characterize the displacement surface and solves the smoothing problem with the Tikhonov regularization [18]. By ignoring the contribution to the matrix from the invalid region, IHFESM could tackle the arbitrarily shaped ROI. Experiments showed that the IHFESM could evidently reduce the noises in the displacement and strain fields obtained directly by Newton–Raphson (NR) iteration. However, IHFESM is still criticized for its high computational complexity, and it could hardly be used to smooth large number of experimental images in practice. The inefficiency comes from finding the optimum regularization parameter within the generalized cross-validation (GCV) function.

* Corresponding author. Tel.: +86 10 62782824 8132; fax: +86 10 62797136.
E-mail address: jqzhao@mail.tsinghua.edu.cn (J. Zhao).

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Each time the GCV function is evaluated, the inverse and the trace of two large scale matrices will be calculated, usually the GCV function needs to be evaluated for 20 times for optimization, and this is the bottleneck of the speed of IHFESM. Besides, the optimum regularization parameter λ varies largely for different problems, and in IHFESM the search range of λ is quite wide, which causes difficulty in optimization and further decreases the accuracy. Another defect of IHFESM is that, the complex two dimensional meshing routines (such as the commercial software ANSYS®) are required to locate and mesh the invalid regions into triangular elements, which makes IHFESM too complicated to be used.

In this work, to settle above issues in IHFESM, we propose the fast Hermite element method (FHEM), which makes improvement from three aspects: (1) FHEM decreases the computation burden by pre-decomposing two small matrices which greatly simplifies the original matrix inverse and trace computation in GCV function. At the same time, when smoothing a group of results of experimental images, the pre-decomposed matrices could be reused and the decomposition needs to be done only once. (2) To narrow down the search range of regularization parameter, the closeness term and roughness penalty term in the error function are balanced with one new parameter before optimization, thus a reliable search range could be assigned and the optimization accuracy is raised. (3) To abandon the complex meshing routines, FHEM computes the global regularization matrix by collecting contributions from the valid points using simple matrix formulation, which enables FHEM to tackle arbitrary ROI. In our experimental study, the proposed FHEM is compared with the original IHFESM, and the result shows that FHEM is at least 50 times as fast as IHFESM with similar accuracy.

The remainder of this paper is organized as follows. In Section 2, the basic principle of DIC, the Hermite finite element, the Tikhonov regularization and the generalized cross-validation are presented. Section 3 demonstrates the details of FHEM. The performance of FHEM is investigated and compared with IHFESM in Section 4. Finally, we end the paper with some conclusions in Section 5.

2. Basic principles

2.1. Digital image correlation

DIC [1,2,19] is a well-established non-contact optical full-field deformation measurement technique, which computes the displacement and strain of POIs in the predefined ROI on speckled surface (see Fig. 1).

To calculate all desired deformation variables, the gray values of pixels in the deformed subset are compared with those of pixels in reference subset. To describe the similarity between the deformed and reference subset, we commonly use the zero-mean normalized cross-correlation criteria (ZNCC) [20], which is defined as

$$\mathbf{C}(\mathbf{p}) = 1 - \frac{\sum_{x=-M}^{M}\sum_{y=-M}^{M}[f(x,y)-f_m][g(x',y')-g_m]}{\sqrt{\sum_{x=-M}^{M}\sum_{y=-M}^{M}[f(x,y)-f_m]^2}\sqrt{\sum_{x=-M}^{M}\sum_{y=-M}^{M}[g(x',y')-g_m]^2}} \quad (1)$$

where $f(x,y)$ represents the gray value of point (x,y) in the reference subset and $g(x',y')$ is the gray value of corresponding point (x',y') in the deformed subset, all the coordinates are depicted in the coordinate system anchored at the center of the reference subset with size $(2M+1)(2M+1)$ pixels, $f_m = \left(1/(2M+1)^2\right)\sum_{x=-M}^{M}\sum_{y=-M}^{M} f(x,y)$ and $g_m = \left(1/(2M+1)^2\right)\sum_{x=-M}^{M}\sum_{y=-M}^{M} g(x',y')$ are the average gray values of all the points in two subsets, and $\mathbf{p}$ is the deformation vector which describes the relationship between coordinate (x,y) and (x',y'). The function value $C(\mathbf{p})$ falls into the range of [0,2], where 0 and 2 indicate the best and the worst match respectively. One merit of ZNCC criteria is that it is insensitive to the scale and offset changes in illumination lighting fluctuations, which often occurs in the practical experiments.

The form of parameter $\mathbf{p}$ depends on the selection of shape function. Generally, when the subset is small enough, the first-order shape function will be used, which maps the coordinate (x,y) to (x',y') by

$$\begin{cases} x' = x + u + \frac{\partial u}{\partial x}x + \frac{\partial u}{\partial y}y \\ y' = y + v + \frac{\partial v}{\partial x}x + \frac{\partial v}{\partial y}y \end{cases} \quad (2)$$

where u and v are the two displacement components of the reference subset center along the x and y directions, $\partial u/\partial x$, $\partial u/\partial y$, $\partial v/\partial x$ and $\partial v/\partial y$ are the four displacement gradient components, thus $\mathbf{p} = [u, \ v, \ \partial u/\partial x, \ \partial u/\partial y, \ \partial v/\partial x, \ \partial v/\partial y]^T$ is the corresponding parameter vector. The first-order shape function allows the translation, rotation, shear, normal strains and their combinations of subset, and it provides all the necessary deformation information for general measurement.

It should be noted that, besides the first order shape function, the second order shape function is also widely used [7,21], and the second order shape function is more suitable for the measurement of deformation of high strain gradient. As stated in [9], the error of DIC results are composed of the systematic error and random error (or random noise), and the undermatched shape function is one of the sources of the systematic error [7], while the image noise is the important factor causing the random noise in results. The purpose of proposed FHEM is to reduce or decrease the random noise in the displacement fields calculated directly by image registration algorithm using either the first order or the second order shape function, and then differentiate the smoothed displacement fields to get smoothed strain fields. Therefore, the proposed FHEM is

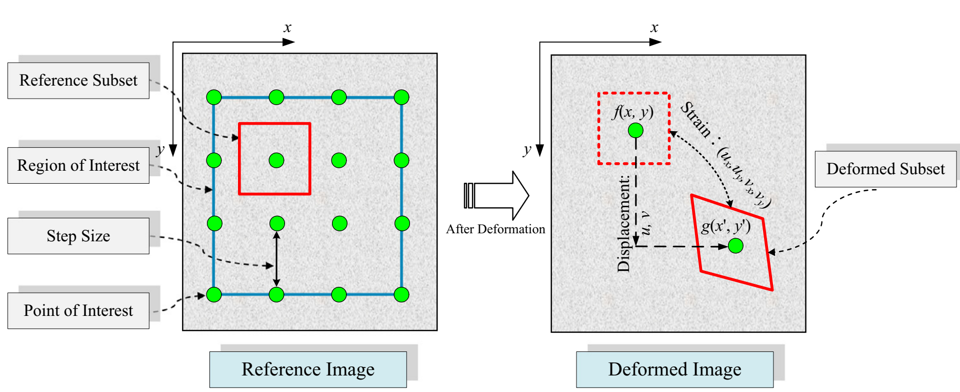

Fig. 1. Definition of the ROI, POI, deformation parameters, reference and deformed subset in DIC.