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Differential 3D shape retrieval

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ABSTRACT

We are presenting a differential three-dimensional (3-D) shape profiling method that is based on the combination of orthogonal fringe projection. It allows us to compute depth gradient maps in a fast and efficient manner. What we are demonstrating is that depth gradients can be computed in a simple way by measuring fringe deformation throughout a novel single-shot approach. We show the usefulness and potential applications of the proposed approach. Validation experiments are presented as well.

1. Introduction

A variety of techniques for three-dimensional (3-D) shape profiling have been proposed in the existing literature. In all the cases, at least two elements are required in order to obtain geometrical information of the scene by means of triangulation. Some methods use two or more cameras (*multi-view stereo*), while other methods use one camera and control illumination sources (*Structured light/Photometric stereo*). Several multi-view stereo methods can be found in the literature (see e.g. Ref. [1] and references therein). Generally they employ multiple cameras placed at various known positions. The different images of the scene then allow corresponding points to be found and depending on the relative positions of those points, depth information can be obtained. The main advantage of those methods is that they do not need flashing lights, and major drawbacks are the need of precise calibration, the fact that textured images are required (to find correspondences) and finally, computation complexity.

Photometric stereo is a simple and traditional 3D retrieval technique first introduced by R. Woodham [2]. The main idea of the method consists in finding the orientation field (normal vectors) of a 3D surface by measuring light reflected when illuminating from different angles. Photometric techniques are sensitive to the presence of projected shadows and ambient illumination. Also, Lambertian surfaces are usually needed.

The basic principle of structured light methods is to project one or more light patterns and extract depth information by measuring the deformation of projected patterns. Numerous techniques for surface imaging by structured light are currently available (see Ref. [3] and references therein for a complete and updated review). These methods can be classified as single shot [4,5] or sequential (multiple-shot). Usually sequential methods produce more reliable and accurate results, but they cannot be used on moving scenes or applications that impose constraint on the acquisition time.

We are presenting a new method for 3D retrieval that can be identified as a single shot, structured light approach. In contrast with most single shot techniques (e.g. Color Coded Stripes, Gray Scale Coded Stripes, De Brujin Sequence, Pseudo Random Binary Dots, Mini-Patterns and Color Coded Grids) [3], the proposed method does not require finding correspondences or matching steps. This is also true for the single shot technique proposed by Takeda and Mutoh [6] where a dense reconstruction of the phase is possible without any matching or calibration steps. However, in contrast with Takeda's method, the proposed technique does not require an unwrapping step.

In the following section, theory and method are described and some advantages of the proposed technique are also listed. In Section 3 we present experimental results, and in Section 4 conclusions are presented.

2. Description of the method

When a scene is being captured by two cameras and assuming that just a translation exists between them, the apparent image shift (disparity) gives information about relative depth of the scene, as illustrated in Fig. 1. Assuming $H \gg \max\{h, f\}$, the disparity $D(x, y)$ between both images (i.e., the shift of the images on the detector arrays of the cameras) and the depth $h(x, y)$ of the test

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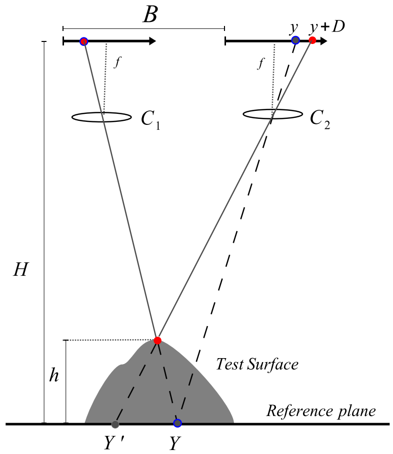

Fig. 1. Principle of stereoscopic vision. The figure shows two cameras $C_{1,2}$ with lenses of focal distance f separated a distance B, placed at a distance H from a reference plane.

surface relative to the reference plane will be related by

$$h \approx \frac{\overline{Y'Y}}{B/H}, \quad \overline{Y'Y} \approx D\frac{H}{f}. \tag{1}$$

the relative errors of the two approximations in Eq. (1) are $(f+h)/(H-f-h)$ and $f/(H-f)$, respectively.

Hence,

$$h \approx D\frac{H^2}{Bf} \tag{2}$$

where f is the focal length, H is the distance of the reference plane and B is the distance between the cameras' centers [7].

The previous expression was obtained by considering two cameras, but it also holds when we have a camera and a projector, as shown in the literature [6,3,8–10]. [For illustrative purpose in Fig. 1 the relative translation between camera and projector is along the y coordinate. In the next, the shift between camera and projector will be along a direction at 45 degrees with respect to the x and y coordinates.]

The procedure for 3D-shape retrieval we are proposing consists in measuring the partial derivatives of the disparity (D), and then, to integrate them with the purpose of retrieving $h(x,y)$ using Eq. (2).

Let us assume that we are projecting a rectangular fringe pattern of period p in the x and y direction over a test surface, where $I(x,y)$ is the pattern acquired by the camera (see Fig. 2(a)). The 2D-spatial spectrum of this image is shown at the center of Fig. 2, in which the red points correspond to the vicinity of the spatial carrier's frequency (i.e. $2\pi/p$). By performing a simple Fourier filtering (red regions in Fig. 2), one can obtain a pattern ($I_v(x,y)$) with deformed vertical fringes and another one ($I_h(x,y)$) with deformed horizontal fringes, as shown in Fig. 2(b) and (c), respectively. [In Fig. 2 we are assuming that the x and y direction are horizontal and vertical, respectively.]

Therefore, it does not matter if the projected fringes are binary or sinusoidal. Without loss of generality, by filtering the spatial spectrum of $I(x,y)$, one obtains

$$I_h(x,y) = I_0(x,y)\ \cos((2\pi/p)(y+D(x,y))) \tag{3}$$

and

$$I_v(x,y) = I_0(x,y)\ \cos((2\pi/p)(x+D(x,y))) \tag{4}$$

where $I_0(x,y)$ is a function of the reflectance of the test surface. As usual, we are assuming that $I_0(x,y)$ and $D(x,y)$ are low-frequency functions in comparison with the frequency of the spatial carrier, i.e. $D_i \ll 1$ and $I_{0i} \ll 2\pi/p$, where the subscript denotes partial derivative with respect to the variable $i(=x,y)$.

Then, by taking partial derivatives with respect to the x and y coordinates, from Eqs. (3) and (4) one obtains

$$I_{hi}(x,y) \approx -(2\pi/p)I_0(x,y)\ \sin[(2\pi/p)(y+D(x,y))](y_i+D_i(x,y)) \tag{5}$$

$$I_{vi}(x,y) \approx -(2\pi/p)I_0(x,y)\ \sin[(2\pi/p)(x+D(x,y))](x_i+D_i(x,y)) \tag{6}$$

where $x_i=1$ and $y_i=0$ for $i=x$, and $x_i=0$, $y_i=1$ for $i=y$.

Hence, it is easy to demonstrate that

$$D_x(x,y) \approx \frac{I_{hx}(x,y)}{I_{hy}(x,y)} \tag{7}$$

and

$$D_y(x,y) \approx \frac{I_{vy}(x,y)}{I_{vx}(x,y)}. \tag{8}$$

We conclude that the gradient of the disparity (D) can be calculated in a simple manner as the ratio of the derivatives of the (horizontal and vertical) Fourier components of the image $I(x,y)$ acquired by the camera.

After obtaining the partial derivatives $\partial D/\partial x$ and $\partial D/\partial y$, we must integrate them to obtain $D(x,y)$ $(=hBf/H^2)$. To avoid confusions due to the notation, let us call the retrieved partial derivatives $\partial D/\partial x = g_x$ and $\partial D/\partial y = g_y$ and the unknown function $D(x,y) = u(x,y)$. The problem of integration [11–13] is equivalent to the problem of finding the function $u(x,y)$ such that the error (or energy function)

$$E[u] = ||\partial u/\partial x - g_x||^2 + ||\partial u/\partial y - g_y||^2$$

is minimized ($||\cdot||$ denotes the standard L-2 norm). The solution we are looking for must satisfy the Euler–Lagrange equation,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial(g_x)}{\partial x} + \frac{\partial(g_y)}{\partial y}. \tag{9}$$

The right side of Eq. (9) can be calculated from the already known g_x and g_y functions. Defining $\partial(g_x)/\partial x + \partial(g_y)/\partial y = G$, the final step consists in solving the Poisson equation,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = G. \tag{10}$$

The solution of Eq. (10) can be achieved in a closed form by using sine or cosine expansions of u and G. For the sake of simplicity, suppose that u and G are of size $L \times L$. Without loss of generality, we can assume that these functions extend outside the interval $L \times L$. Specifically, we will assume that they are periodic $2L$ (in both directions) and odd in the variables x and y. Thus, we can write

$$G(x,y) = \sum_{k=1}^{k=L}\sum_{n=1}^{n=L} d_{nk}\ \sin\left(\frac{\pi kx}{L}\right)\ \sin\left(\frac{\pi nx}{L}\right) \tag{11}$$

$$u(x,y) = \sum_{k=1}^{k=L}\sum_{n=1}^{n=L} c_{nk}\ \sin\left(\frac{\pi kx}{L}\right)\ \sin\left(\frac{\pi nx}{L}\right). \tag{12}$$

It is easy to see that, in order to satisfy Eq. (10), c_{nk} must be

$$c_{nk} = -\left(\frac{L}{\pi}\right)^2 \frac{d_{nk}}{n^2+k^2}. \tag{13}$$

Finally, by substituting Eq. (13) into Eq. (12) the depth map can be obtained as desired.

3. Comparison with other methods

The procedure for 3D-shape retrieval described in the present paper has various advantages with respect to other fringe projection methods presented in previous literature. Firstly, image partial derivatives can be computed fast and efficiently. Secondly,