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# An efficient and accurate 3D displacements tracking strategy for digital volume correlation

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ABSTRACT

Owing to its inherent computational complexity, practical implementation of digital volume correlation (DVC) for internal displacement and strain mapping faces important challenges in improving its computational efficiency. In this work, an efficient and accurate 3D displacement tracking strategy is proposed for fast DVC calculation. The efficiency advantage is achieved by using three improvements. First, to eliminate the need of updating Hessian matrix in each iteration, an efficient 3D inverse compositional Gauss–Newton (3D IC-GN) algorithm is introduced to replace existing forward additive algorithms for accurate sub-voxel displacement registration. Second, to ensure the 3D IC-GN algorithm that converges accurately and rapidly and avoid time-consuming integer-voxel displacement searching, a generalized reliability-guided displacement tracking strategy is designed to transfer accurate and complete initial guess of deformation for each calculation point from its computed neighbors. Third, to avoid the repeated computation of sub-voxel intensity interpolation coefficients, an interpolation coefficient lookup table is established for tricubic interpolation. The computational complexity of the proposed fast DVC and the existing typical DVC algorithms are first analyzed quantitatively according to necessary arithmetic operations. Then, numerical tests are performed to verify the performance of the fast DVC algorithm in terms of measurement accuracy and computational efficiency. The experimental results indicate that, compared with the existing DVC algorithm, the presented fast DVC algorithm produces similar precision and slightly higher accuracy at a substantially reduced computational cost.


## 1. Introduction

As a novel and useful tool for quantifying the internal 3D deformation across the entire volume of opaque solids and biological tissues, digital volume correlation (DVC) has gained increasing attention and found various applications in the fields of biomechanics [1–3], experimental mechanics [4,5], material research [6–8] and biomedical engineering [9,10]. DVC was originally developed by Bay et al. in 1999 [11] to investigate the strain maps throughout the trabecular bone. As a 3D extension of its 2D counterpart, which is known as 2D digital image correlation (2D-DIC) [12], the basic principle of DVC is equally simple and straightforward. In brief, DVC tracks the relative movements of reference sub-volumes defined in the reference volume image in deformed volumetric images to retrieve the 3D displacement vector field. Afterwards the displacement fields are differentiated using a proper numerical differentiation approach to obtain full-field strain maps.

However, compared to its 2D counterpart, the inherent computational cost of DVC is much heavier, roughly cubically higher than that of 2D-DIC. This is because cubic sub-volumes, rather than square subsets, are tracked in the deformed volumetric images in DVC. Further, the region of interest defined in DVC is extended to a spatial cubic volume. Suppose 2D-DIC calculation is performed on a grid of $K \times K$ points using a $(2M+1) \times (2M+1)$ pixels subset, a similar DVC calculation will be implemented on a grid of $K \times K \times K$ points with a $(2M+1) \times (2M+1) \times (2M+1)$ voxels sub-volume. As such, the computation cost of DVC is $(2M+1) \times K$ times higher than that of 2D-DIC. For instance, if the subset (subvolume) size $M$ is chosen to be 10 pixels (voxels) and $K$ is selected as 100 points, the computational cost of DVC will be at least 2100 times heavier than that of 2D-DIC. Considering the growing trend of using higher-resolution volumetric images and the frequent need of processing a sequence of volumetric images recorded in different configurations, high-efficiency and high-accuracy DVC method has become increasingly important. It has been demonstrated that the computational efficiency of DVC can be enhanced by fully exploring the advanced parallel computing capability of modern computers with multicore processors [13]. The main focus of this work, however, is on the DVC algorithm.

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To retrieve the displacement vectors with sub-voxel accuracy, conventional DVC algorithms [11,14–16] generally adopt a two-step calculation strategy. First, integer-voxel displacements of the reference sub-volume are determined using a simple spatial-domain [15] or frequency-domain [4] searching scheme within the deformed volume image. Then, using the obtained integer-voxel displacements as initial estimate, a sub-voxel registration algorithm, combining a non-linear iterative optimization algorithm (e.g., the Gauss–Newton algorithm) and a sub-voxel intensity interpolation algorithm (e.g., the tricubic interpolation), is employed to calculate displacements with sub-voxel accuracy. Although conventional two-step DVC techniques perform well with acceptable measurement accuracy in most cases, one evident drawback of the traditional DVC method, however, remains to be its extremely huge computational cost. The reason why the traditional DVC calculation is computational expensive is owing to the following three aspects.

First, it is noted that nearly all the sub-voxel registration algorithms used in DVC, involving the Gauss–Newton algorithm [11] proposed by Bay et al., the BFGS Quasi-Newton algorithm [14] presented by Verhulp et al., and iterative least-squares algorithm [16] advocated by Pan et al., employ an easy-to-understand forward additive matching strategy. One common feature of these algorithms is that the pre-assumed target sub-volume deformed by an initial guess of deformation parameters is compared with the original cubic reference sub-volume to estimate an additive increment to the deformation parameters, which is subsequently added to the initial estimate to update the parameters. Although these forward additive algorithms feature simple principle, easy implementation and high measurement accuracy, an inherent limitation of these algorithms is that the Hessian matrix must be recalculated in each iteration. The repeated computation of Hessian matrix for every calculation point at each round of iteration greatly increases the computational cost of DVC calculation. For this reason, an iterative algorithm eliminating the need of updating Hessian matrix is advantageous in improving the efficiency of DVC calculation.

Second, almost all the sub-voxel registration algorithms, which are essentially local non-linear optimization algorithms, require an initial estimate of deformation parameters that are sufficiently close to the true values to converge accurately and rapidly. The closer the initial estimate is, the faster the convergence speed will be. Despite that integer-voxel displacement searching can be carried out efficiently using the sum-table approach [15,17] or fast Fourier transform for each measurement point [4], it still consumes more or less computation time. Consequently, a one-step calculation strategy free of integer-voxel displacement search should obviously be more efficient.

Third, during each step of iterative optimization using conventional algorithms, certain sub-voxel interpolation algorithm must be repeatedly used to reconstruct the intensity as well as intensity gradients at each sub-voxel location for the displaced voxel points of target subvolume. Note that the sub-voxel interpolation calculation of a voxel point of certain reference sub-volume is not only performed in each round of iteration, but also needs to be carried out for the same voxel point appeared in adjacent reference sub-volumes, since the interrogated sub-volumes defined in the reference volume image are normally highly overlapping. The repeated interpolation calculation performed at sub-voxel locations, in particular, consumes most redundant calculations of the traditional DVC method. Because of this, how to avoid the redundant calculation in sub-voxel interpolation becomes a primary challenge in efficient DVC calculation.

In this work, to eliminate all the aforementioned redundant calculations involved in existing DVC calculation, an efficient and accurate 3D displacement tracking strategy, employing three principle improvements, is proposed. First, an efficient 3D inverse compositional matching scheme combined with the Gauss–Newton algorithm (hereafter referred to as 3D IC-GN algorithm), initially proposed by Baker et al. [18], is introduced to optimize the practical zero-mean normalized sum of square difference (ZNSSD) criterion for accurate 3D sub-voxel displacement measurement. In Ref. [19], inverse compositional algorithm has been proved equivalent to the forward additive algorithms for 2D image alignment. However, as will be shown later, compared with the traditional forward additive Gauss–Newton (FA-GN) algorithms, the inverse compositional algorithm is more efficient. Second, the proposed DVC employs a one-step calculation strategy and thus avoids the need of integer-voxel displacement searching. Instead, the accurate and complete initial estimate is automatically transferred from its processed neighbors using an ingenious and robust reliability-guided displacement tracking strategy, which warrants a faster convergence speed of the IC-GN algorithm. Thus, repeated and time-consuming integer-voxel displacement searching is eliminated. Finally, an interpolation coefficient look-up table approach is utilized for avoiding the redundant calculations involved in the process of tricubic interpolation. Through these improvements, the proposed DVC method can achieve obvious speed advantages over the existing DVC method without losing the computational accuracy. The accuracy and efficiency of the proposed DVC method is demonstrated by processing numerically translated 3D speckle patterns.

## 2. Fast DVC using 3D inverse compositional Gauss–Newton algorithm

### *2.1. 3D inverse compositional Gauss–Newton algorithm for sub-voxel registration*

The basic principle of the 3D IC-GN algorithm for sub-voxel displacement tracking is schematically shown in Fig. 1. First, to quantitatively evaluate the similarity between the reference and deformed sub-volumes, the robust ZNSSD criterion [20,21], combined with the practical affine transform warping function, is employed in this work.

$$C_{ZNSSD}(\Delta \mathbf{p}) = \sum_{\xi} \left\{ \frac{f(\mathbf{x}+\mathbf{W}(\xi;\Delta \mathbf{p}))-f_m}{\Delta f} - \frac{g(\mathbf{x}+\mathbf{W}(\xi;\mathbf{p}))-g_m}{\Delta g} \right\}^2 \tag{1}$$

where f($\mathbf{x}$) and g($\mathbf{x}$) are the gray values at point $\mathbf{x}$ within the reference and deformed volume images; $\xi=(\Delta x, \Delta y, \Delta z, 1)^T$ is the local coordinates of a voxel point in the sub-volume; $f_m$ and $g_m$ are the mean intensity values of the reference and deformed sub-volumes; $\Delta f = \sqrt{\sum_{\xi} [f(\mathbf{x}+\mathbf{W}(\xi;\Delta \mathbf{p}))-f_m]^2}$ and $\Delta g = \sqrt{\sum_{\xi} [g(\mathbf{x}+\mathbf{W}(\xi;\mathbf{p}))-g_m]^2}$

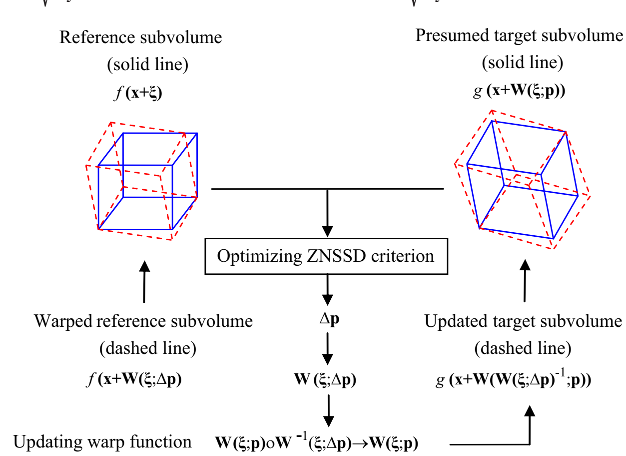


**Fig. 1.** Schematic illustration of the principle of the 3D IC-GN algorithm.