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A model of inbound air traffic: The application to Heathrow airport

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A B S T R A C T

We present a model to describe the inbound air traffic over a congested hub and we show that this model gives a very accurate description of the traffic by comparing our theoretical distribution of the queue with the actual distribution observed at Heathrow airport. We also discuss the robustness of our model.

1. Introduction

Airport congestion is a persistent phenomenon in air traffic. Air traffic congestion is significant even if the principal airports in Western and Central Europe are treated as "fully coordinated",[1] meaning essentially that the number of flights that can be scheduled per hour (or other unit of time) is not allowed to exceed the "declared capacity" of the airports (de Neufville and Odoni, 2003). In 2011, the average ASMA additional time[2] at the top 30 european airports amounted to 2.9 min per arrival, increasing by +5% with respect to the previous year. London Heathrow is a clear outlier, having by far the highest level of additional time within the last 40 nautical miles (NM) with 8.2 min per arrival, followed by Frankfurt and Madrid (EUROCONTROL, 2012). Similar situations occur in the US (Ball et al., 2001).

Several approaches have been proposed to mitigate congestion and resolve demand-capacity imbalances. At an operational level (short-term) these approaches consider the operational adjustment of air traffic flows to match available capacity. So far, the most popular approach in resolving these short-term periods of congestion has shown to be the allocation of ground delays (Odoni, 1987). The Ground Holding Problem considers the development of strategies for allocating ground delays to aircraft, and it has received considerable attention (Andreatta et al., 2011; Ball et al., 2010; Dell'Olmo and Lulli, 2003; Richetta and Odoni, 1994). However, these air traffic flow management strategies might be suboptimal because they do not capture the inherent unpredictability of arrivals at airports. Willemain et al. (2004) showed that changes in the current practice for setting airport arrival rates can lead to significant benefits in terms of additional ASMA times.

In view of the current situation, it is extremely important to have a reliable tool to measure and forecast congestion in the air traffic system. However, in developing such a tool there are some issues to address. First of all, the stochastic models developed so far to describe air traffic congestion are not reliable. Willemain et al. (2004) showed that the estimated inter-arrival times at a distance of 100NM from the final destination are nearly exponential. In other words, the arrival stream can be considered Poissonian when entering the control zone.[3] The aircraft stream is successively rearranged to meet the air traffic control (ATC) rules and needs, so it is natural to expect that Poissonian arrivals will give a poor fit with the actual arrivals at a congested airport.[4] Nevertheless, Poissonian arrivals are very often considered as the actual arrival stream when studying actual scenarios (Balakrishnan and Chandran, 2006; Bäuerle et al., 2007; Dunlay, 1976; Marianov and Serra, 2003).

A second issue regards the validation of the stochastic models. It is not easy to draw a comparison between observed and forecast

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[1] In the U.S., scheduling limits are applied only to New York region airports, Washington/Reagan, and Chicago/O'Hare airport, under the High Density Rule (HDR).

[2] The Arrival Metering and Sequencing Area (ASMA) is the airspace within a radius of 40NM around an airport. The ASMA additional time is a proxy for the average arrival runway queueing time of the inbound traffic flow, during times when the airport is congested.

[3] With respect to any London airport, the control zone is a large area covering England and Wales, operated by the London Area Control Centre (LACC). See Section 3 for more details.

[4] Cf. Fig. 5a below.

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congestion, simply because it is hard to retrieve the number of aircraft in the queue from a database of flights. Indeed, it is not clear a priori which procedure should be used to extract information about the airport congestion from a data set of waypoints passage times. In particular, the problem we address is how to determine the correct fraction of time each aircraft actually spends in the queue.

In this paper we propose a possible answer to both the aforementioned problems. Regarding the former, in Section 2 we present a description of the arrival pattern and build a mathematical model for the queue at the airport. The latter issue is addressed by computing the time spent in queue by the aircraft as follows. Given a terminal route and a database of flight records, the minimum time lapse between the entrance in the control zone and the touch-down is subtracted from the time flown by any other aircraft in the terminal route. In this way the time spent in one or more stacks by the aircraft is found. A detailed discussion of this approach is given in Section 3. Eventually, the analysis is completed by comparing

- the stationary output of the mathematical queueing model defined in Section 2;
- the distribution of the queue obtained from a data set of arrivals at London Heathrow airport.

In particular, we will see in Section 4 that the fit of the actual Heathrow data with the output of the proposed model is very close, and very much better with respect to the output of an equivalent queueing model assuming Poissonian arrivals.

In fact, we will see in Section 2 that the proposed description of the arrival pattern gives rise to a family of point processes. Each of these are obtained from a deterministic schedule stream by superimposing independent and identically distributed (i.i.d.) random delays. The list of actual times of arrival is then the result of the mixing-up of the fixed schedule by the random delays. The process obtained in this way has a long history (Kendall, 1964). It is easy to study numerically but quite difficult to treat from a mathematical point of view, though significant progress has been recently made (Guadagni et al., 2012, 2011). In Section 5 we will show by numerical simulations that the output of the model depends only in a very weak manner on the kind of delays added to each arrival time. In other words, the Probability Density Function (PDF) of the random variables added to the deterministic arrival stream has a very small impact on the distribution of the observed queue. The only relevant parameter appears to be the variance of the random delays. Therefore, any PDF is suitable and a reliable forecast of the traffic over a congested hub one can also use very simple random variables (e.g. uniform).

2. Description of arrival process and queueing model

Let $1/\lambda$ be the expected inter-arrival time between two consecutive aircraft. The Pre-Scheduled Random Arrivals (PSRA) is an arrival process such that the actual time of arrival of the i-th aircraft is

$$t_i = \frac{i}{\lambda} + \xi_i, \quad i \in \mathbb{Z}, \tag{1}$$

where ξ_i are real, continuous, i.i.d. random variables with compact support. The delays ξ_i have finite variance σ^2, their PDF is denoted by $f_\xi^{(\sigma)}(t)$. Without loss of generality we can assume $\mathbb{E}(\xi_i) = 0$, as $\mathbb{E}(\xi_i) \neq 0$ only affects the initial configuration of the system. The expectation of the arrival time of the i-th customer is then $\mathbb{E}(t_i) = i$.

When σ is large the process defined in (1) weakly converges to the Poisson process, in particular it is possible to prove that its generating function tends point-wise to the generating function of the Poisson process (Guadagni et al., 2011). This property also holds for a variant of the PSRA process that takes into account the possibility of the flights' cancellation, as in Ball et al. (2001). This variant is an independent thinning version of this process, i.e., a process in which each arrival has an independent probability $1-\gamma$ to be cancelled (and the complementary probability γ to be a *true* arrival). Fig. 1 shows the output of a thinned PSRA, the actual stream of aircraft (ATA) is the consequence of the random delays ξ_i mixing-up the pre-scheduled, expected stream (ETA). In the Air Traffic Management (ATM) context it is natural to couple a PSRA arrival process with a deterministic service process with expected service rate λ. Since the inter-arrival rate of the thinned process is $\gamma\rho$, then the traffic intensity of the queueing model is clearly $\rho = \gamma$.

Remark 1. According to the model description above, the traffic load ρ must be intended as a parameter of the model itself. In 2007 the airport of London Heathrow operated at an actual flow rate between 97 and 98% of its runway capacity (SH&E Limited, 2008), this sets $0.97 \leq \rho \leq 0.98$ for our model. In Section 3 we will see an operative definition of ρ based on actual data.

The queue of this model, n_t, is then a well-defined, continuous-time stochastic process. Associated to this process we can consider the embedded chain, i.e., the length of the queue at the service epochs. The latter is a discrete-time Markov chain defined by the recursion

$$n_{k+1} = n_k + m_{(k,k+1]} - \left(1 - \delta_{n_k,0}\right), \quad k \in \mathbb{N}, \tag{2}$$

where $\delta_{i,j}$ is the usual Kronecker's delta and $m_{(k,\,k+1]}$ is the number of arrivals in the interval $(k,\,k+1]$, that is, the number of those aircraft $\{i_1, i_2, \ldots, i_S\}$ such that their actual arrival time is $t_{i_s} \in (k, k+1]$, $s = 1, \ldots, S$. The presence of the Kronecker's delta ensures that the queue at time $k+1$ is decreased by one unity only if the queue length at time k is positive. After Kendall's notation, such a queueing model will be hereafter called PSRA/D, as in Gwiggner (2011), Nikoleris and Hansen (2012).

Remark 2. PSRA/D is a mathematical model of a queueing system. As such it may be far from modelling a real system with respect to some specific features. For example, there are no explicit flight separation rules as they are implicitly accounted for by the deterministic service time with fixed duration $1/\lambda$. Nonetheless, such a plain vanilla model is capable to deliver a very accurate goodness of fit with actual data, as we will see in Section 4. Therefore it does make sense to study it, and successively try and improve it to include more features of real ATM/Air Traffic Control (ATC) systems.

Although for large σ both the PSRA process and its thinned version are very similar to the Poisson process, they present a crucial difference with the latter. Whenever σ remains finite, the PSRA process is negatively autocorrelated. The covariance Cov(n_1, n_2) between the number of arrivals at two consecutive time periods n_1 and n_2, where n_1 is the number of arrivals in $(t, t+T]$ and n_2 is the number of arrivals in $(t+T, t+2T]$, is given by

$$\begin{aligned}\mathrm{Cov}(n_1, n_2) &= \mathbb{E}(n_1 n_2) - \mathbb{E}(n_1)\mathbb{E}(n_2) \\ &= -\sum_i p_i^{(\sigma)}(t, t+T)\, p_i^{(\sigma)}(t+T, t+2T),\end{aligned}$$

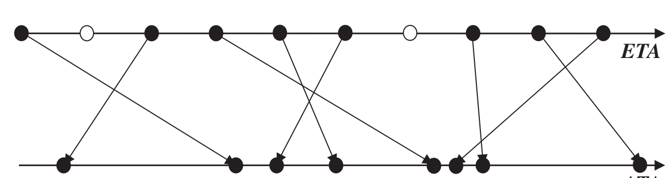

Fig. 1. The actual stream of arrivals (ATA) rises from the action of random delays and thinning (white dots) on a deterministic schedule (ETA).