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Carrier fringes interferometry by superposing the first harmonic of two rulings with different period

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ABSTRACT

This paper presents a method for introducing carrier fringes in an interferogram based on the interference of first harmonic of two rulings with different periods. The setup is built on a 4*f* optical system consisting of two apertures in the input plane and a Ronchi ruling in the Fourier plane; in each aperture a Ronchi ruling of a different period is placed. This proposal is based on filtering in the Fourier plane the 1st-order of diffraction of each spectrum of the rulings in the object plane; we demonstrate that the slope of the generated linear phase depends directly on period difference. In this manuscript, we develop a theoretical model and show experimental results.

1. Introduction

Carrier fringe interferometry (CFI) [1–4] consists of introducing a linear phase term into an interferogram, whose carrier frequency is greater than the absolute maximum frequency in the interferogram. Thus it consists of open fringes and its Fourier transform consists of three separated lobules, in such a form that one of them can be isolated from the others. Next, once that this filtered lobule is centred, and the inverse Fourier-transform is applied, then the object phase can be obtained. Thus, this method is able to retrieve the object phase from only one interferogram. However, the major experimental difficulty and the main error source is in the generator of the linear phase since the existing methods to introduce carrier fringes need, for example, to tilt a mirror by a very small angle [4–6], to use a wedge prism [7] also with a very small angle, or recently in a form more convenient, by moving a Ronchi ruling outside from the Fourier plane [8,9]. However this method needs very fine displacements and besides, an error in its angular position will introduce an error in the phase.

To overcome these drawbacks, in this manuscript, we present a new idea to introduce carrier fringes, which is based on superposing the first harmonic of two Ronchi rulings of different periods, whose difference is proportional to the phase slope introduced. We show that an efficient form to obtain a wide range of ruling periods is produced by turning azimuthally two Ronchi rulings having the same period. The present proposal has two important advantages, the first advantage consists of using the Ronchi ruling at the input plane instead of at the Fourier plane for introducing carrier fringes [8,9] since as it very well-known the position at the Fourier plane is more critical than the input plane because an error in the ruling position will be greater for the ruling placed at the Fourier plane. Thus for instance, the position of the ruling at the entrance can be placed outside of an object plane, without appreciable phase changes in the interferogram. The second important advantage consists of turning the ruling angles within the range $(0, \pi/3)$ radians, which are very large compared with the mirror tilt angles or apex wedge that need angles of milliradians (mrad) [6,7]. Because these angles are very large compared with those in the mirror tilt or in a wedge prism, this proposed method is more practical and considerably improves the accuracy. Additionally, in the present proposal, the beam is not deviated as in the methods that use a tilted mirror or wedge prism.

2. Theoretical analysis

The present proposal is based on a double aperture common-path interferometer (DACPI) depicted in Fig. 1. It consists of two apertures w_s in the input plane, with $s = o,r$, and a Ronchi ruling $\tilde{R}_{\mathfrak{J}}$ of period ν_p in the Fourier plane. An aperture w_o supports a phase object and the other one w_r is left free in order to obtain the probe

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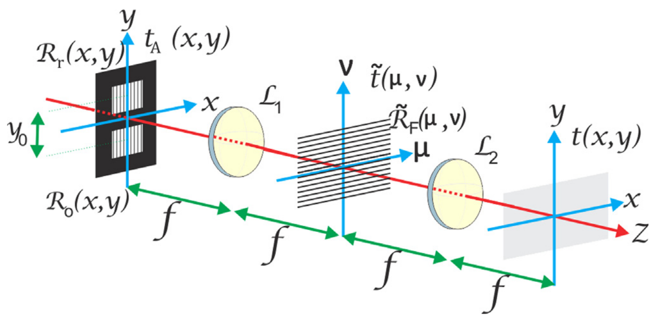

Fig. 1. Experimental setup based on double-aperture common-path interferometer: R Ronchi ruling, L lens, and f focal length.

and reference beams, and additionally, in each aperture a Ronchi ruling R_s of period x_{sp} is also placed. If the setup is illuminated with a non-tilt wave plane, homogeneous, monochromatic with wavelength λ, linearly polarized, coherent, and traveling on z-direction, the input optical field is described by

$$t_A(x,y) = w_o(x,y)R_o(x,y) + w_r(x,y)R_r(x,y) \tag{1}$$

where w_s is yielded by

$$w_s(x,y) = w(x,y-y_s)A_s(x,y-y_s)\exp[i\phi_s(x,y-y_s)] \tag{2}$$

$w(x,y)=rect(x/a_w)rect(x/b_w)$ is a window of sides a_w and b_w with rect(…) denoting the rectangle function, $-y_o = y_r = y_0/2$ with y_0 denoting of vertical separation of the windows. A and ϕ are the amplitude and phase of the fields that pass through the apertures $w(x,y-y_s)$. On the other hand, the rulings described in the complex Fourier series are given by

$$R_s(x,y) = \frac{x_{sw}}{x_{sp}}\sum_k \sin c\left(\frac{x_{sw}}{x_{sp}}k\right)\exp\left(i2\pi\frac{k}{x_{sp}}x\right) \tag{3}$$

where x_{sw} is its brilliant bar. In the Fourier plane the optical field is described by the Fourier-transform of the entrance field given in Eq. (1), in other words

$$\tilde{t}_A(\mu,\nu) = \tilde{w}_o(\mu,\nu) \otimes\otimes \tilde{R}_o(\mu,\nu) + \tilde{w}_r(\mu,\nu) \otimes\otimes \tilde{R}_r(\mu,\nu) \tag{4}$$

where $\tilde{w}_s$ and $\tilde{R}_s$ are the Fourier-transform of w_s and R_s, respectively, $(\mu,\nu)=(u/\lambda f, v/\lambda f)$ are the frequency coordinates, f is the focal length of the lenses, and $\otimes\otimes$ denotes the convolution operator in two dimensions,

$$\tilde{R}_s(\mu,\nu) = \frac{x_{sw}}{x_{sp}}\sum_k \sin c\left(\frac{x_{sw}}{x_{sp}}k\right)\delta\left(\mu-\frac{k}{x_{sp}},\nu\right) \tag{5}$$

then, by substituting Eq. (5) into Eq. (4), we have

$$\tilde{t}_A(\mu,\nu) = \sum_k\left\{c_{ok}\tilde{w}_o\left(\mu-\frac{k}{x_{op}},\nu\right) + c_{rk}\tilde{w}_r\left(\mu-\frac{k}{x_{rp}},\nu\right)\right\} \tag{6a}$$

where

$$c_{sk} = \frac{x_{sw}}{x_{sp}} \sin c\left(\frac{x_{sw}}{x_{sp}}k\right) \tag{6b}$$

thus, the optical field in the Fourier plane consists of replicas of $\tilde{w}_s$ separated by $u_{sp} = \lambda f/x_{sp}$. If the frequency components of $\tilde{w}_s$ are small enough so that they do not overlap, then each order k can be isolated. In frequency coordinates the filter to be used is described

$$\tilde{f}_{\mathfrak{F}}(\mu,\nu) = rect(\bar{x}_p\mu - k) \tag{7}$$

where $\bar{x}_p = (x_{op}+x_{rp})/2$, and analytically this filtering operation can be defined as $\tilde{t}_A(\mu,\nu)\tilde{f}_{\mathfrak{F}}(\mu,\nu) = \tilde{t}_{Ak}(\mu,\nu)$, and it is

$$\tilde{t}_{Ak}(\mu,\nu) = c_{ok}\tilde{w}_o\left(\mu-\frac{k}{x_{op}},\nu\right) + c_{rk}\tilde{w}_r\left(\mu-\frac{k}{x_{rp}},\nu\right) \tag{8}$$

the kth harmonic of series given in Eq. (6a,b). Finally, the field after it crosses the ruling placed in the Fourier plane is given by

$$\tilde{t}'_{Ak}(\mu,\nu) = \tilde{t}_A(\mu,\nu)\tilde{f}_{\mathfrak{F}}(\mu,\nu)\tilde{R}_{\mathfrak{F}}(\mu,\nu) \tag{9}$$

where, in the complex Fourier series, the ruling is written

$$\tilde{R}_{\mathfrak{F}}(\mu,\nu) = \frac{\nu_w}{\nu_p}\sum_n \sin c\left(\frac{\nu_w}{\nu_p}n\right)\exp\left(i2n\pi\frac{\lambda f}{\nu_p}\nu\right) \tag{10}$$

ν_w is its brilliant bar. At the image plane, the optical field is given by the inverse Fourier-transform of the expression described in Eq. (9), that is

$$t_k(x,y) = t_{Ak}(x,y) \otimes\otimes R_{\mathfrak{F}}(x,y) \tag{11}$$

where $t_{Ak}(x,y) = \mathfrak{F}^{-1}\left\{\tilde{t}_A(\mu,\nu)\tilde{f}_{\mathfrak{F}}(\mu,\nu)\right\}$, or explicitly

$$t_{Ak}(x,y) = c_{ok}w_o(x,y)\exp\left(i2\pi\frac{k}{x_{op}}x\right) + c_{rk}w_r(x,y)\exp\left(i2\pi\frac{k}{x_{rp}}x\right) \tag{12}$$

which is a modified entrance function that consists of the same entrance apertures modulated by a factor c_{sk}, whose magnitude depends on its fill factor, and besides both apertures have a factor of linear phase whose slope depends on the filtered order k and its period x_{sp}. On the other hand, $R_{\mathfrak{F}}(x,y) = \mathfrak{F}^{-1}\left\{\tilde{R}_{\mathfrak{F}}(\mu,\nu)\right\}$, or explicitly

$$R_{\mathfrak{F}}(x,y) = \sum_n d_n\delta\left(x, y-\frac{\lambda f}{\nu_p}n\right) \tag{13a}$$

where

$$d_n = \frac{\nu_w}{\nu_p} \sin c\left(\frac{\nu_w}{\nu_p}n\right) \tag{13b}$$

Then, by substituting Eq. (13a) into Eq. (11) the output optical field can be described by,

$$t_k(x,y) = \sum_n d_n t_{Ak}\left(x, y-\frac{\lambda f}{\nu_p}n\right) \tag{14}$$

which consists of replicas of the modified entrance optical field, as shown in Eq. (12), whose distance of separation is given by $\lambda f/\nu_p$. Then, by substituting sequentially Eq. (12) and Eq. (2) into Eq. (14), after many manipulations the followings expression can be obtained

$$t_k(x,y) = \sum_n w\left(x, y-\left(n+\frac{1}{2}\right)y_0\right)\left[c_{ok}d_{n+1}A_o\exp(i\phi_o)\exp\left(i2\pi\frac{k}{x_{op}}x\right) + c_{rk}d_nA_r\exp(i\phi_r)\exp\left(i2\pi\frac{k}{x_{rp}}x\right)\right] \tag{15}$$

where the matched condition $y_0 = \lambda f/\nu_p$ has been assumed, and the spatial coordinates for A_s and ϕ_s have been omitted. On the other hand, because in practice an optical detector observes only a rectangular region of the form $w(x, y-(n+1/2)y_0)$, this observation can be described by a filtering operation of Eq. (15) given by $w(x, y-(n+1/2)y_0)t_k(x,y)$, and in other words,

$$t_{k,n}(x,y) = c_{ok}d_{n+1}A_o\exp(i\phi_o)\exp\left(i2\pi\frac{k}{x_{op}}x\right) + c_{rk}d_nA_r\exp(i\phi_r)\exp\left(i2\pi\frac{k}{x_{rp}}x\right) \tag{16}$$

A practical case occurs when $k=1$ and $n=0$, then, omitting coordinates, the irradiance is described as,

$$I = c_{o1}^2d_1^2A_o^2 + c_{r1}^2d_0^2A_r^2 + 2c_{o1}c_{r1}d_0d_1A_oA_r\cos(\phi+\alpha_{pR}) \tag{17a}$$

where $\phi=\phi_o-\phi_r$ is considered the object phase, and $\alpha_{pR}=\alpha_{pR}(x)$ is a term of linear phase given by,

$$\alpha_{pR}(x) = 2\pi\mu_{pR}x \text{ with } \mu_{pR} = \frac{1}{x_{pR}} = \frac{1}{x_{op}} - \frac{1}{x_{rp}} \tag{17b}$$

Eq. (17a) is an interferogram with carrier fringes, and μ_{pR} is known as a carrier frequency, which is a function of the periods x_{sp}