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Generation of optical vortices by fractional derivative

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ABSTRACT

This paper presents a new method of vortex generation using two-dimensional fractional derivative. The characteristics of vortices obtained using this method from Gaussian and Hermite–Gauss distributions are presented. Changing the parameters of fractional derivative such as the fractional order, r, and the direction, θ, the positions of the vortex centers can be changed. The method can be used to design a filter for vortex generation. The analysis of an experimental vortex pattern using fractional derivative is also demonstrated.

1. Introduction

In recent years, fractional calculus has proved a useful mathematical tool in a number of branches of physics and engineering [1,2]. It is interesting to see how fractional calculus can be used as a usual mathematical tool in optics. For example, [3] explored the possibility of extending the fractional derivative to the fractionalization of a Gaussian beam.

A new thresholding technique based on two-dimensional fractional differentiation is developed to be used in image analysis for the identification of the regions of image objects and extraction of the objects from their background [4]. To improve pattern recognition, a new optical system that combines a fractional derivative function with correlation between objects was suggested [5]. A recent paper also demonstrated the applicability of fractional derivative (FD) to the phase retrieval from the fringe pattern [6].

A large number of papers are devoted to the generation of optical vortices and their applications. Some review articles present the generation of optical beams having vortices, their behavior and applications [7–10]. Now, there are various ways of generating optical vortices such as: (1) combination of cylindrical lens [11], (2) a spiral phase plate [12–14], (3) a computer-generated hologram using liquid crystal spatial light modulator [15,16], (4) a q-plate [17], (5) forked polarization gratings [18], (6) interferometric techniques [19], (7) uniaxial crystal [20], (8) Fresnel lens with embedded vortices [21], etc.

A light beam, which propagates in the oz direction, is represented in a cross-sectional plane by a complex function

$$f(x,y) = A(x,y)\cdot \exp[i\varphi(x,y)] = \mathrm{Re}(f) + i\cdot \mathrm{Im}(f)$$

where $A(x,y)$, $\varphi(x,y)$, $\mathrm{Re}(f)$ and $\mathrm{Im}(f)$ are the amplitude, phase, real and imaginary parts of the field $f(x,y)$, respectively. This beam possesses an optical vortex only if there is at least one point where the amplitude is zero and the phase is undefined. In the central point of the vortex the following relations are true: $\mathrm{Re}(f)=0$ and $\mathrm{Im}(f)=0$. These conditions can be easily obtained by applying the fractional derivative operator to the field described by $f(x,y)$ because FD acts like a filter in Fourier plane.

In this paper we present a method of vortex generation based on FD. The definition and characteristics of the FD used are presented in Section 2. Two examples of vortex generation from Gaussian and Hermite–Gaussian beams are presented in Sections 3 and 4. Sections 5 and 6 deal with the regular lattice generation of optical vortices.

2. Fractional derivative of order *r* in a given direction

There are several definitions for fractional derivative of a one-dimensional function $f(x)$. In image processing domain it is common to use a definition based on Fourier transform. In the case of one-dimensional function $f(x)$, fractional derivative of order r (r being a real number) is given by

$$D^r f(x) = FT^{-1}\lfloor (i\omega_x)^r FT(f(x))\rfloor \qquad (1)$$

where FT and FT^{-1} are the Fourier transform and inverse Fourier transform respectively of the one-dimensional function $f(x)$ and $\omega_x = 2\pi\nu_x$ with ν_x being the variable in frequency domain.

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For a two-dimensional function, $f(x, y)$, a fractional derivative of order r in a given direction, θ, is calculated in relation to coordinates ω_x and ω_y by:

$$D_\theta^r f(x,y) = FT2^{-1}[M_r \cdot FT2(f(x,y))] \tag{2}$$

where $FT2$ and $\mathrm{FT2}^{-1}$ are, respectively, two-dimensional Fourier transform and its inverse for the function $f(x, y)$ and $M_r(\omega_x, \omega_y)$ is a notation for

$$M_r(\omega_x, \omega_y) = (i\omega_x \cos\theta + i\omega_y \sin\theta)^r \tag{3}$$

The function $M_r(\omega_x, \omega_y)$ is defined in frequency space and acts like a filter in this space.

For special cases, $\theta=0$ or $\pi/2$, Eq. (2) becomes the one-dimensional fractional derivative corresponding to the ox and oy directions, respectively, with the same expression as in Eq. (1).

Eq. (2) represents the function itself for the order $r=0$, and the sum of first order partial derivative with respect to x and y for $r=1$.

It can be observed that $M_r(\omega_x, \omega_y)$ is a complex function characterized by an amplitude and phase distribution in Fourier space. In the discrete form, the function $M_r(\omega_x, \omega_y)$ is an $M \times N$ complex matrix, where M and N are the maximum numbers of the mesh points in ωx and ωy directions, respectively.

In the calculus, the matrix of $M_r(\omega_x, \omega_y)$ is centered on $(M/2, N/2)$ and has the following form:

$$M_r(\omega_x, \omega_y) = [i(\omega_x - M/2)\cos\theta + i(\omega_y - N/2)\sin\theta]^r \tag{4}$$

Fig. 1a and c presents the amplitude and the phase of matrix from Eq. (4) for $M=N=512$.

Fig. 1a shows the normalized amplitude of matrix $M_r(\omega_x, \omega_y)$ having a minimum value of zero for the coordinates corresponding to the phase step. The variation of the amplitude is nonlinear with respect to ω_x and its curvature depends of the value of r. The amplitude variation is linear only for the first-order derivative corresponding to $r=1$, as can be seen from Fig. 1b.

Fig. 1c shows that the phase has a step of $2r(\pi/2)$ to the middle of the map phase. The direction of the phase step in different phase maps is given by the value of θ, representing the rotational angle of the phase step around the center of the image. The sections from Fig. 1d show that the step phase is from $-r(\pi/2)$ to $r(\pi/2)$.

The profile of the normalized amplitude remains the same as in Fig. 1a to the changing of the direction θ, but it rotates itself around the center of the image. The map phase shows the same step phase as $2r(\pi/2)$, but it is rotated with an angle, θ.

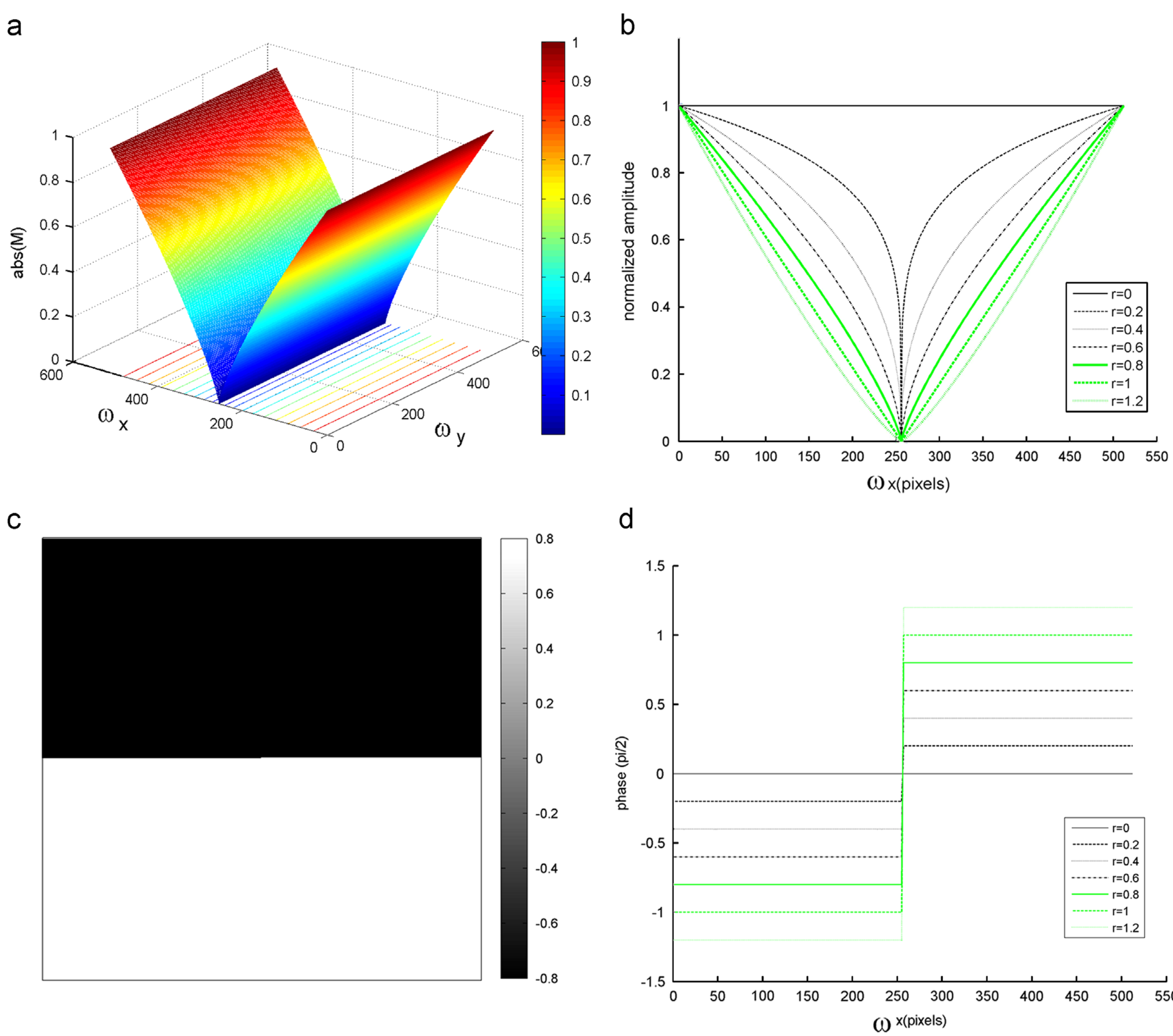

Fig. 1. Amplitude and the phase of matrix $M_r(\omega_x, \omega_y)$ from Eq. (4): (a) normalized amplitude for $r=0.8$ and $\theta=\pi/2$; (b) sections of normalized amplitude at $\omega y=256$, $r=0$, 0.2, 0.4, 0.6, 0.8, 1, 1.2 and $\theta=\pi/2$; (c) the map of phase for $r=0.8$ and $\theta=\pi/2$; and (d) sections of phase maps at $\omega y=256$, $r=0$, 0.2, 0.4, 0.6, 0.8, 1, 1.2 and $\theta=\pi/2$.